Abstract

Local search is one of the most successful approaches for solving hard optimization problems. In local search, a set of neighbor solutions is assigned to every solution and one asks for a *local optimum*, i.e., a solution that has no better neighbor. The neighborhood relation between the solutions naturally induces *standard algorithms* that find a local optimum: Begin with a feasible solution and iteratively move to a better neighbor until a local optimum is found. Many empirical and theoretical investigations have shown that these methods quickly terminate in a local optimum for most instances.

For some problems, however, instances were found for which a standard algorithm *can* take an exponential number of improving steps if the initial solution and the rule that chooses among the improving neighbors, i.e., the *pivot rule*, are unluckily chosen. Even worse, for some problems, instances and initial solutions were found in which, independent of the pivot rule, *every* standard algorithm takes an exponential number of steps. We say that these problems have the *all-exp* property. Thus, using a standard algorithm turns out to be impractical in some cases.

But how hard is computing a local optimum then—using standard algorithms or any other approach? To encapsulate the complexity of finding local optima, Johnson et al. (JCSS,1988) introduced the complexity class PLS. Shortly afterwards, Schäffer et al. (JOC,1991) showed PLS-completeness for several local search problems including LocalMax-Cut on graphs with unbounded degree with a FLIP-neighborhood in which one node changes the partition. Moreover, they showed two further results for LocalMax-Cut: It has the all-exp property and the StandardAlgorithmProblem (SAP), i.e., the problem of finding a local optimum that is reachable from a given pair of an instance and initial solution via a standard algorithm, is PSPACE-complete. On the positive side, Poljak (JOC,1995) showed that there are at most $O(n^2)$ improving steps possible for LocalMax-Cut on cubic graphs. He also posed the question whether it has the all-exp property on graphs with maximum degree four. Due to the huge gap between the degree three and an unbounded degree, Ackermann et al. (JACM,2008) asked for the smallest $d \in \mathbb{N}$ for which LocalMax-Cut on graphs with maximum degree d is PLS-complete.

This thesis provides three complexity results for LocalMax-Cut. First, it has the all-exp property if restricted to graphs with maximum degree four—this solves the problem stated by Poljak. Second, the SAP is PSPACE-complete for graphs with maximum degree four. Third, finding a local optimum is PLS-complete for graphs with maximum degree five—this solves the problem of Ackermann et al. almost completely since d is narrowed down to four or five (unless PLS $\subseteq$ P). Since LocalMax-Cut has been the basis for several PLS-reductions in the literature, the results have impact on further problems. Some of the reductions directly carry over the degree in some way and transfer the complexity results to the corresponding problems even for very restricted sets of feasible inputs.