Essays on Investment Decisions under Large Uncertainty

Dissertation

zur Erlangung des akademischen Grades
des Doktors der Wirtschaftswissenschaften (Dr. rer. pol.),
eingereicht an der Fakultät für Wirtschaftswissenschaften
der Universität Paderborn im Februar 2014

vorgelegt von:

Natasa Bilkic, Dipl.-Math., M. Sc.

Gutachter:

1. Prof. Dr. Thomas Gries
2. Prof. Dr. Wim Naudé

Acknowledgments

I would like to take this opportunity to show my deepest gratefulness to all those people who have supported and accompanied me on my way so far. First of all I would like to thank my advisor, Prof. Dr. Thomas Gries, without whom I would not be able to write this dissertation. His excellent professional skills, guidance, steady encouragement and generous support showed me how exciting research can be. From the first day on, working with him was characterized by fruitful and deep going discussions that steadily improved my work.

I am also thankful to my scientific advisors Prof. Dr. Wim Naudé (Maastricht School of Management) and Prof. Dr. Rick van der Ploeg (University of Oxford) for their guidance and giving me the opportunity to participate in joint projects. It was a great pleasure to work with such inspiring academic professionals.

Additionally, I am thankful to my colleagues at the Chair of International Growth and Business Cycle Theory for their help and kind words. Each and every created a great working atmosphere and every day with them was enjoyable and stimulating.

Especially many thanks to Denis Klinac, Johann Rudi, Margarete Redlin and Stefan Gravemeyer for proofreading this thesis. The remaining mistakes are mine.

I would also like to thank my parents for their constant love and support. Finally my deepest gratitude goes to my fiancé Valeri Lorenz, whose love encouraged and inspired me. He went with me through good and bad and always believed in me. He and my family provided me with strength and made me to the person who I am. Without them I would not be able to go this way, which has not always been easy.

Contents

List of Tables

List of Figures

Chapter 1

Introduction and Comprehensive Summary

> *"It is a world of change in which we live, and a world of uncertainty. We live only by knowing something about the future; while the problems of life, or of conduct at least, arise from the fact that we know so little. This is as true of business as of other spheres of activity."*

Frank H. Knight (1921)

The financial crisis, which started in 2008, has shown that economic events of great severity are difficult to predict. Until then, the complete failure of banks had not been likely and the financial market had been assumed to recover without massive interference of governments. This opinion changed drastically after the bankruptcy of the Lehman Brothers bank especially because many people and institutions were hit by enormous losses. The resulting financial crisis has also shown that financial institutions and investors were not able to judge the riskiness of their investments, or at least, they were not able to manage it. Henceforward, economic agents wanted to be better prepared for every contingency in order to react adequately, what in turn increased the attention of professionals to reconsider investment behaviour in an uncertain environment. In this context the following questions arise: What is uncertainty? Is uncertainty desirable? How can uncertainty be evaluated and included in investment decisions? What are the effects of uncertainty on investment behaviour?

The concept of uncertainty is not new but even though it is something that all of us face every day, it is difficult to describe. Frank H. Knight (1921) was one of the first to take a deeper look into this topic and to distinguish risk from uncertainty. While risk describes a situation where the parameters of the probability distribution of possible future outcomes can be determined, for describing uncertainty no statements about probabilities can be issued. In other words, risk describes an uncertain situation that can be converted into an effective certainty. Real uncertainty, instead, is intangible. Furthermore, Knight argues that the analysis of uncertainty goes back to consciousness itself because the survival of humans depends on their forward-looking character. Hence, humans have to adapt to a situation before it happens. In order to do so, the presupposition that the world is made up of things, which, under the same circumstances, always behave in the same way, is required (see Knight, 1921). For instance, it is generally known that apples should be ripe by the beginning of autumn. Depending on the weather and especially on how sunny the summer was, the final date can vary. If a farmer wants to plant a new apple tree and use the apples to produce juice, he cannot be exactly sure about when his own apples will be ripe. But from the experience of other farmers he will expect the harvest not before September. As we can see from this example, expectations about future developments are formed based on their own past or based on the past of similar things. This view does not, however, imply that everything is exactly predictable, it only says that we can come close to it by assigning probabilities to particular outcomes. Since the path-breaking work of Knight (1921) the distinction between risk and uncertainty has been established in economics and finance; however, uncertainty remained difficult to account for.

Economic activity takes place in an uncertain environment where uncertainty can have different forms. Figure 1·1 illustrates the path of the Standard and Poor's stock index of the 500 largest firms in the US between 2005 and 2013. This index is often used as a performance indicator of those firms and the US economy. The figure shows that on the one hand the index has systematic and expected movements. That is, it has a business cycle with economic booms and downturns as well as a growth structure. Before the financial crisis in 2008 there was an increasing path which started to decrease due to large losses suffered by the considered firms during the recession after 2008. The economy started to recover in 2009 even with a larger volatility. In addition, Figure 1·1 shows a small but usual variability in the S&P index due to changing economic activity, seasonal conditions etc. This variability represents a form of uncertainty that can be measured either by the variance, if a random variable at one point in time is considered, or by the volatility of a stochastic process, if the focus lies on the evolution of a random variable. Even more striking is the unexpected larger variability from one moment to the next that occurs as a consequence of economic, political or ecological shocks. For instance, natural disasters such as the hurricanes Katrina and Sandy caused a sudden and drastic downward movement in the stock index and therewith a lower performance of the considered firms. A similar picture can be found after influential political events such as the US presidential election in November 2008. At that time, the economic performance of the US increased due to optimistic expectations, so the stock index experienced a large upward movement. Large scale movements in the stock index can also result from important news such as the Standard and Poor's downgrade of the US from AAA to AA+ on 8th August 2011. This day made history as the "Black Monday" because the Dow Jones dropped by 5,5%, which is the 6th largest drop in the history of the index.

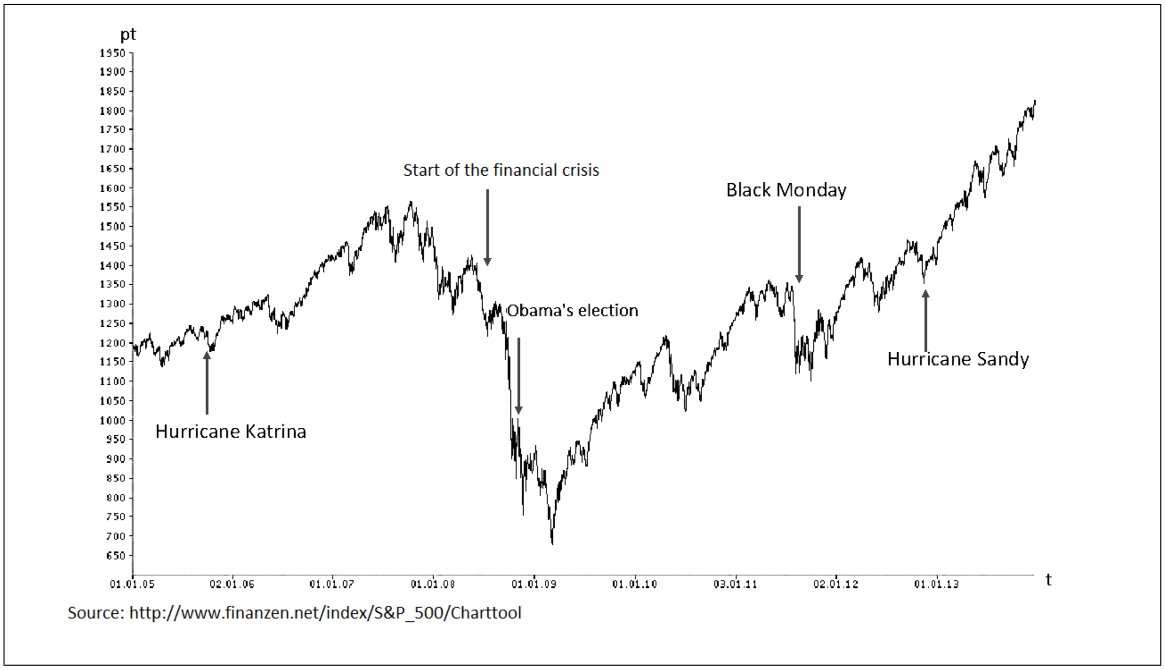

Figure 1·1: S&P 500 Stock Index from 01.01.2005 to 24.12.2013

From this discussion it can be concluded that uncertainty is a phenomenon that is generated by random deviations from initially expected developments and that uncertainty has two forms. On the one hand, there are marginal and in some sense expected random shocks from one moment to the next due to usual variability in the economy. On the other, there are non-marginal stochastic shocks as a consequence of massive changes in economic, political and ecological conditions that refer to disasters or opportunities.

As non-marginal stochastic events appear to determine the economic performance of firms and therewith of countries, they may determine investment decisions of individuals as well. This doctoral thesis sheds further light into the mathematical and economic modelling of large variability such as disasters and analyse their effect on investment decisions. It is shown that large variability is an essential element of investment decisions and must not be neglected in the evaluation of investment projects. The next section will give a brief overview about how such stochastic variability has been evaluated so far and lead over to the focus of this doctoral thesis.

1.1 Evaluation of Variability

Every person reacts differently in risky situations.[1] Risk-loving people take every opportunity to experience risk and they are willing to pay for it, even if their live is in danger. For instance, a base-jumper risks his live to experience the ultimate adrenaline rush. The utility function of such a risk-loving agent is always convex because more risk is connected to a higher utility. Other people are more cautious and try to avoid any risky situation. If those so-called risk-averse people have to take a risk, they want to be rewarded somehow. For instance, a flight attendant has a risky job so that in addition to the wage he or she obtains a danger bonus, which can be interpreted as a risk premium. In this case, the utility function is concave because larger risk reduces the utility of a risk-averse agent. In contrast to both groups mentioned, there are also people that are completely risk-neutral and have a linear utility function. Hence, risk does not affect their utility, so they neither pay nor expect to be rewarded for their risky action. A measure of the degree of risk aversion goes back to Arrow (1971) and Pratt (1966) who introduced the Arrow-Pratt-coefficient η

$$\eta(x) = -\frac{U''(x)}{U'(x)},$$

[1] See, e.g., Kruschwitz (2011), pp. 293.

with U being a twice differentiable, monotonically increasing utility function. For a risk-neutral agent the Arrow-Pratt coefficient η is 0, for a risk-averse agent it is positive and for a risk-loving agent it is negative. η is a coefficient which determines the premium that an agent obtains or the payment that he or she pays for taking the risk. Hence, it is a parameter that increases or decreases the return of an investment project depending on the risk attitude of the agent. As a consequence, taking risk and uncertainty indeed can be desirable because an investor can obtain additional returns if risky investments are undertaken.

The next question of how uncertainty and risk can be evaluated for investment decisions is not simple to answer. For the inclusion of risk, however, we need to know what kind of investment decision is considered. The literature distinguishes static, dynamic and sequential investment decisions that can be formed in discontinuous or continuous time.[2] Static optimization assumes that the investor only considers the outcome and not when the outcome is obtained, while dynamic approaches discount future returns in order to account for the rationality of the investor. Since static approaches are not in the focus of this doctoral thesis, it is abstained from further explanations. An often used example of a dynamic investment approach is the Expected Net Present Value (ENPV) method, which also is a starting point of this doctoral thesis. The second chapter introduces an extension of the ENPV to account for the possibility of disasters. However, if the investment decision is irreversible, meaning that the costs of investment are sunk, then sequential decisions, that allow the determination of an investment strategy over time, become relevant. More precisely, at every point in time the investor decides about the best investment opportunity: to invest, not to invest at all or to postpone the decision to a later date. The set of those investment decisions refers to sequential investment decisions (see

[2] See, e.g., Kruschwitz (2011) and Dixit and Pindyck (1994).

Kruschwitz, 2011, pp. 299). Besides obtaining the best investment strategy, sequential approaches can also make statements about the optimal time of investment.

These kinds of decisions are picked up in the third and fourth chapter, where a decision rule, as well as the optimal timing of investment decisions, is determined within a real option framework and dynamic programming. Furthermore, both chapters allow for the flexibility to postpone the decision about the irreversible action to a later date in order to collect more relevant information that may affect the desirability or the timing of investment. The inclusion of such flexibility is of major importance because, as Dixit and Pindyck (1994) and Trigeorgis (1996) argue, neglecting it can result in large losses. In any case, the investor decides about a set of mutually exclusive investment opportunities. The optimal choice is then given by the investment opportunity that maximizes the satisfaction of the decision maker, while satisfaction is defined by a preference functional given by the utility or the return on investment.

Risk can be included into the above approaches in various ways. The simplest method is to account for the variability around an expected value by implementing the variance of the respective random variable. In a static approach, investment decisions are based on the expected return and the variance. Markowitz (1952) was the first to apply the so-called Mean-Variance approach to the portfolio selection of risk-averse agents and his contribution is considered as a major step in the theory of modern finance.[3] During the following decades, the Mean-Variance approach has been extended to capture Roy's idea of loss aversion (Roy, 2001). That is, investors are assumed to be less concerned about obtaining more, than about avoiding losses. First attempts in the evaluation of losses were introduced by more sophisticated concepts such as the semi-variance and its generalizations (see, e.g., Fabozzi and

[3] See, e.g., Fabozzi et al. (2007).

Markowitz, 2002; Unser, 2002), and Value at Risk (Chow and Kritzman, 2002). One major characteristic of these approaches is that they are all based on an ex-post analysis of existing data. Hence, the evolution of the value in question is assumed to be similar to the past. Furthermore, these approaches do not distinguish marginal and non-marginal stochastic shocks. An alternative concept to the Mean-Variance analysis was provided by the Stochastic Dominance principle (Hadar and Russels, 1969) where probability distributions of asset values are compared. Hence, an investor is in favour of the asset, that has a lower cumulative distribution function, hence, where low values have a low probability.

In dynamic or sequential frameworks, where the evolution of a random variable is of particular importance, the volatility of the stochastic process is considered. Most notably, Black and Scholes (1973) use the geometric Brownian motion to describe the evolution of an asset value and derive the price for its option. They also argue that variability, measured by the volatility of the respective stochastic process, increases the price of the option. Hence, taking a higher variability may be beneficial for the investor because a higher variability may increase the price of the asset and therefore result in a larger price of the respective option. Since their very influential contribution, this stochastic process became the workhorse process in finance and related fields. Although the modelling introduced by Black and Scholes (1973) was a benchmark for many following applications, Merton (1975) emphasizes the inability of the Brownian motion to capture real characteristics of asset prices and instead promotes the use of jump processes. He argues that most of the time an asset follows a Brownian motion and, with a known probability, jumps by random amplitude. With his argumentation, Merton motivated the reconsideration of the prevailing evaluation methods, even beyond the option pricing theory. Hence, Merton was the first to emphasize that a new view of variability is necessary and that not only marginal variability, as modelled by volatilities, should be considered. Based on this argument, Cox et al. (1976, 1979) discuss different jump processes and distinguish between discrete and continuous time where the jump probability is always known. Almost 20 years later, Amin (1993) extends the discrete modelling of Cox et al. (1979) by including simple jump diffusion processes for American options. He finds that discrete time models converge to their corresponding continuous time models and shows that jumps can significantly alter the optimal exercise decision. A more general model is proposed by Pham (1997), who considers a process that contains a geometric Brownian motion and simultaneously an accumulation of jumps with random jump amplitude, and compares the obtained option value of an American option to an European option. In his approach, the jump amplitude and the time of a jump are random. The importance of jump diffusion processes is revisited by Kou (2002). He argues that because of a higher peak, two heavier tails and a volatility smile in asset prices, an evaluation model should allow for jumps. By assuming a double exponential distribution for the jump amplitude in a general jump diffusion model, he derives the price for an European option. Kou and Wang (2003a) and Bayraktar and Xing (2008) find an approximation for the American option price, and Kou and Wang (2003b) solve optimal stopping problems including the first passage time for American options. Abundo (2010) extends their model by a random threshold determined by a transformed compound Poisson process.

However, especially after the increasing number of economic, political, and natural catastrophes the importance of the evaluation of disasters and their inclusion in decision making has already been recognized by empirical investigations in different contexts. For instance, Brancati (2007) and Berrebi and Oswald (2011) argue that natural disasters, as a form of large and influential uncertainties, may lead to more conflict outbreaks in countries that have weak institutions. Gries and Meierrieks (2013) show that financial crises, as an example of economic disasters, have the identical consequence. A theoretical modelling of disasters is provided by Cox et al. (2000, 2004) and Kousky et al. (2010), who argue that jump diffusion processes are useful for modelling catastrophes. Since this phenomenon is especially relevant in insurance contexts, Yang and Zhang (2005) and Jang (2007) apply the model of Cox et al. (2000, 2004) to derive optimal investments of insurance companies and moments of losses. The inclusion of large variability in theoretical contributions can also be found in some financial applications, e.g. by Wachter (2013), where a more general view of risk and uncertainty is used to explain the equity premium puzzle.

Although the introduced literature is a first step to include large variability into investment decisions, a transfer to general project evaluation and their consequences for decision-making has not taken place yet.

1.2 Large Uncertainty and Risk in this Thesis

The following three chapters revisit and extend the evaluation of variability in economic decision making and apply the developed approaches to different economic problems. It is emphasized how marginal and non-marginal stochastic shocks have an opposed effect on investment decisions and that neglecting them may lead to huge losses.

In a nutshell, the thesis starts with a dynamic approach and discusses the simplest evaluation method, namely the Expected Net Present Value technique (ENPV). In this case, an investment is undertaken if the expected present value of the project is larger than its costs. The purpose of this procedure is to start with a simple model and to show the consequences of large stochastic shocks for economic decision-making. That is, such events, although being rare, are identified to be important elements of economic and project reality but are not included in common evaluation methods like in the ENPV. Furthermore, usually it cannot be stated when and with which severity the next overthrowing event will occur. Therefore, a more complex stochastics has to be implemented to account for them in investment decisions. In order to capture those characteristics, the ENPV technique is extended by using the Ito-Lévy Jump Diffusion process that can model disasters occurring at uncertain points in time with a large uncertain damage. It is shown that this extension is necessary because only accounting for marginal shocks, such as modelled by the geometric Brownian motion, may lead to non-profitable investment decisions. This type of modelling goes beyond simple statements about risky events, where probabilities of events with a particular impact can be assigned, so that it comes closer to Knight's notion of uncertainty.

Later, this simple view is extended to sequential investments, where the timing of an irreversible investment is the focus. Hence, we are not only interested in whether to take the risk, rather in, when taking the risk. The optimal decision is determined by comparing investment costs with benefits, that are obtained when investing immediately, and with benefits, that may be obtained when investing later. Hence, it is a decision about waiting and not waiting longer to invest. In the first extension, large stochastic variability happens to occur only in the realization of the project, while in the second, it is also present during the waiting period. The decision problem is then divided into subproblems that are solved statically and recursively with dynamic programming.

The next chapter, *Investment under Threat of Disaster*, is a joint work with Thomas Gries and is a slightly revised version of the paper that was published as a working paper No. 2014-04 in the CIE Working Paper Series. An earlier version of the paper was presented at the Second World Congress of the Public Choice Societies

(Miami, 2012). The chapter starts with a discussion of the relevant literature that considers the evaluation of disasters and points out that the recent increased number of economic and political disasters, such as the financial and economic crisis, and terrorist attacks, motivated the revision of current evaluation methods. While some attempts have been done after Merton's critique (1975) on the geometric Brownian motion, a transfer to simple evaluation methods of investment projects has not taken place yet. Furthermore, this contribution departs from methods that use simple probabilistic statements about risk. That is, an approach is proposed, which assumes that the probability of the next disaster of a particular size is not known. Starting with a simple model, this contribution provides an extension of the ENPV, because it is a starting point of a number of more general evaluation methods which are for instance used in the field of sequential optimization. It belongs to the class of dynamic methods and discounts future returns of the investment project in order to account for the fact that immediate returns are preferred rather than future returns. For the modelling framework, an investor, that determines the profitability of a risky project, is considered. The investor evaluates the project by calculating the expected value of the future discounted returns and by comparing it with the investment costs. The major goal of this contribution is to show that in a highly uncertain environment where disasters are likely to occur, it is not sufficient to only consider marginal stochastic shocks, e.g. modelled by the volatility of a geometric Brownian motion. In other words, neglecting non-marginal stochastic shocks may even lead to non-profitable investment behaviour and enormous losses. In order to show these drastic effects, the ENPV is first derived for a project whose returns follow a geometric Brownian motion. In the next step, the stochastic process is extended by accounting for disasters that occur at an uncertain point in time with an uncertain effect on the project value. This so-called Ito-Lévy Jump Diffusion process, which was also used in Pham (1997) and Kou and Wang (2003a,b), is a more general version of the geometric Brownian motion. In particular, it extends the geometric Brownian motion by adding a term for non-marginal downward jumps. The comparison of the two ENPVs obtained with the two different stochastic processes shows that stochastic variability has an opposed effect: if the project value is calculated with the geometric Brownian motion, the ENPV only contains a discount factor that depends on the interest rate and the drift parameter. That is, the volatility, which is the measure of the marginal stochastic shocks in the stochastic process, is not part of the ENPV and therefore marginal variability does not determine the decision of the investor. If not only marginal variability is considered but also a measure of non-marginal stochastic shocks, the discount factor in the ENPV again depends on the interest rate and the drift parameter. Beyond that, it also contains the frequency and severity of disasters. Hence, these additional components, which include characteristics of disasters, are part of the discount parameter that the investor obtains for suffering from non-marginal stochastic shocks. This result implies the conclusion that agents do not account for marginal shocks but they do for non-marginal disasters when using ENPV as an evaluation method. In other words, the realization of the investment project strongly depends on the degree and severity of the variability of the project value. So far, the effects of such variability have not been considered in the simple ENPV technique. They occur only if we have large and non-marginal shocks.

In addition, this chapter provides a discussion of the different effects of stochastic variability on the investment decision, while the effects are derived from the derivatives of the ENPV with respect to the volatility and the disaster variable. It is pointed out that a larger frequency of disasters lowers the project value while (marginal) volatility has no effect. To illustrate the impact of disasters, an example is simulated. In particular, parameters such as investment costs, interest and average growth rates are held constant and the jump parameters such as the intensity and severity are doubled stepwise. The results show that doubling both parameters, frequency and shock size, decreases the value of the project by more than 30 %, so that the project value turns out to be even negative with a greater frequency and severity of disasters. In contrast, increasing marginal risk does not change the project value.

To sum up, this contribution builds a starting point for the inclusion of disasters in economic decision-making. Furthermore, it points out that in dynamic investment models stochastic variability should not only be measured by marginal stochastic shocks, rather, non-marginal shocks, such as disasters, should also be accounted for. Disregarding this kind of variability results in misleading decision rules, in the sense that non-beneficial projects are treated as beneficial. Hence, the decision maker is likely to bear huge losses.

Chapter 3 is based on the paper *Stay in School or Start Working? - The Human Capital Investment Decision under Uncertainty and Irreversibility*, which was published in 2012 in Labour Economics 19 (5), pp. 706-717. It is a joint work with Thomas Gries and Margarethe Pilichowski, and earlier versions of it were presented at several conferences such as at the Annual Conference of the Canadian Economic Association (Quebec, 2010) or at the 25th Annual Congress of the European Economic Association (Glasgow, 2010). This contribution adds to the literature in two different ways. On the one hand, it provides a new view on the education decision by modelling it as an irreversible investment project under large risk and therewith explaining some important characteristics which will be explained later. On the other, a methodological progress has been made by including large stochastic shocks into the real option framework and determining the income threshold that education needs to generate as well as the expected optimal time to leave school.

In particular, that chapter extends the contribution of Hogan and Walker (2007) and starts with building up a real option framework that includes the major components of the education decision. First, education is a process that may take decades. A student attends primary, secondary and probably tertiary education and pays for every period some amount of money for teaching materials, tuition fees and private lessons. These costs accumulate over time while during the schooling period the student does not obtain any earnings from professional employment. Second, after the student has left school, he or she enters the job market and obtains a reward for his or her work effort. His or her working life generates an individual earning profile, with an entry-level wage and steepness determined by the years of schooling attended and the degree obtained. That is, the entry-level wage, the first wage obtained after entering the labour market, increases with the years of schooling. Beginning with the insights of Mincer in the 1960s, it is a stylized fact that education increases the productivity of a worker and this in turn increases his income. The evolution of the complete earning profile is affected by education as well because more education involves a steeper and a more volatile income structure. For instance, a university graduate might become an influential business manager with an annual income of many million Euros, or he or she might work as a waiter and live below the poverty line. The expected earning profile is assumed to be positively affected by education, especially when an education degree such as the A-levels or a university degree are successfully completed. The additional income connected to the achievement of a degree results in a non-marginal positive income premium, which is called a sheepskin effect. Yet, the income profile may also be negatively affected by negative events such as unemployment, disease or short time work, which was introduced in many companies in Germany after the financial and economic crisis in 2008. Hence, although the income profile is assumed to increase with education, there are major uncertainties during working life, which have to be taken into account for the education decision. Third, further education has a value and staying in school for a little while longer may be more beneficial than irreversibly leaving the education system and entering the labour market. For instance, leaving school after four years at a university may already involve a high income profile but finishing the degree may lead to an extra premium that in turn may lead to an even higher income profile. Accordingly, postponing the labour market entry may provide an additional value in that all immediate and future potential expected income profiles can be compared and the best one can be chosen.

The methodological novelties of this contribution are provided by the innovative inclusion of large stochastic opportunities and threats to the career, which are modelled as major uncertain events, and by the analytical solution to the optimal expected time of schooling. The paper starts with introducing the real option framework that includes all the components described before. In the next step, the particular entry-level wage that is necessary for the student to enter the labour market is determined. This is done by using the major advantage of the real option framework: the sequential comparison of the net wealth of education (Expected Net Present Value of human capital) and the value generated by the flexibility to postpone the labour market entry (value of waiting). The resulting so-called income threshold is the wage that optimally compensates the student for paying the education costs as well as the wage that makes education profitable. Furthermore, it is the optimal earning profile that the student can expect with his education. Please note that as long as the threshold has not been reached it is better for the student to gain more education and improve the expected income track.

After having determined the income threshold, the expected optimal time of schooling is computed. This first hitting time can be interpreted as the point in time at which the labour market entry is expected to happen. The labour market entry may be affected by a set of variables. The paper argues that a larger volatility of the earning profile and higher education costs per period increase the time of schooling, while a larger drift of the income profile as well as a larger no-education income level lead to less education.

The initial model of the human capital investment decision is extended by introducing sheepskin effects that shift the income profile to a higher level after a particular education degree has been achieved. Formally speaking, sheepskin effects are uncertain income premiums that represent discontinuities in the earnings profile. Other opportunities and threats connected to the career that lead to a non-marginal jump in the earning profile are also included in this approach. In order to show the decision mechanism of a student, a sequence of decisions is considered while the earning profile is now described by an Ito-Lévy Jump Diffusion process. In this sense, two income thresholds for each education level are determined by backward calculation. Besides the formal progress that has been made with the inclusion of uncertain events causing a non-marginal jump in the income profile, the paper proofs the strong relationship between sheepskin effects and educational achievements. In particular, it can be argued that larger income premiums extend schooling and lead to a higher investment volume.

Although this paper is an improvement in human capital investment theory, it also provides a methodological extension of the first paper of this doctoral thesis. The paper *Investment under Threat of Disaster* was a first step to show the interrelationship of large variability and investment decisions with one of the simplest evaluation methods. Therefore, it builds a starting point for the second one. In particular, a similar version of the obtained ENPV can be found in the second paper as one of the building blocks of the sequential investment decision. A further extension is that two periods are considered. The first period is characterized by accumulating education costs without any financial compensation, while the start of the second period stops the investment process and leads over to an exclusive earning process. Since education involves a decision process that takes place at each point in time, education can be terminated immediately or extended for one more period in order to postpone the decision to a later date. This sequence of decisions characterizes a dynamic as well as sequential investment behaviour that is based on the option value of waiting introduced by the real option framework. Hence, at each point in time, the ENPV is compared to the value of waiting and an equilibrium can be found where the two are equal. Furthermore, the required compensation to leave school, the optimal time of market-entry as well as the total investment volume are determined. Finally, this contribution provides a suggestion of how non-marginal stochastic shocks, such as disasters or opportunities, can be included in an investment evaluation approach where timing is of particular importance. As already seen in the first contribution, the effect of stochastic variability on the investment decision is two-fold. While the expectation of a more volatile income profile extends the time of schooling, more opportunities provided by the labour market cause the student leave school earlier. For this reason this paper can be used in many other investment problems of a similar structure, such as investments in R&D or innovations.

Chapter 4 presents the last paper of this doctoral thesis, *Uncertainty and Conflict Decision*. This paper is a joint work with Thomas Gries which was published as a working paper No. 2014-05 in the CIE Working Paper Series. Furthermore, it is a revised version of the paper *When to Attack an Oppressive Government,* which was published in the Conference Paper Series 2012 of the Verein für Socialpolitik as "Beiträge zur Jahrestagung des Vereins für Socialpolitik 2012: Neue Wege und Herausforderungen für den Arbeitsmarkt des 21. Jahrhunderts - Session: Conflict and Disputes, No. C20-V3". Earlier versions were presented at several conferences such as at the European Meeting of the Econometric Societies (Malaga, 2012), the World Congress of the Public Choice Societies (Miami, 2012) or at the Australasian Meeting of the Econometric Societies (Melbourne, 2012).

This paper applies the real option framework to the decision to start a social conflict, e.g. a terrorist attack, a riot or an assassination of a political leader. The basic idea of the model is as follows: In a society a particular (minority) group is discriminated by another group or by the government, so that the living conditions of the affected members are expected to worsen over time. For instance, the government may discriminate those members by adopting repressing policies, arresting some members or by refusing political, economic and social participation. The group neither knows when the next repressive action will take place nor does it know how it will be affected. Individuals that belong to a group may become increasingly frustrated by their situation so that they want to change their conditions somehow, and if there is no other way even with launching an attack. We can think of many examples in the past, where conflicts emerged as a consequence of government's repressive actions. For instance, the Arab spring, which started in 2010, aimed to terminate discrimination and repression by the government as well as to increase satisfaction among the people in those countries that did not belong to the rich political elite.

A conflict can be regarded as an investment in a better future and in a change of the political, social and economic conditions. The major problem a rebel faces are the decisions about whether it is beneficial enough and if so when to attack. This decision is made at every moment and is based on the comparison of uncertain future benefits, costs and the value of postponing the decision to a later date. It is therefore dynamic and sequential, so that it can be approached by the real option framework and dynamic programming.

Before and after attacking, the rebel has an expectation about the potential benefits of conflict, which are described by an Ito-Lévy Jump Diffusion Process respectively. The major advantage of such an approach is that unexpected non-marginal stochastic shocks, e.g. generated by the government, as well as expected marginal variations of the living conditions are included in the decision. The focus of this contribution is, however, on uncertain events that occur before the conflict because we want to show how uncertain events may cause an investment.

Although this contribution has a similar methodological structure as the third chapter of this doctoral thesis, it departs in that non-marginal stochastic shocks are now present in both periods and not only in the second. The option value of a later attack is included into the decision because waiting has a value. Before attacking, the rebel can collect relevant information and postpone the irreversible action to a later date. Furthermore, waiting may open up additional action alternatives indicating that a later attack is even more beneficial than an immediate one. Or while waiting, the rebel may realize that the latent conflict has resolved without violent means so that an attack becomes useless. By comparing the option value of a later attack with the ENPV of conflict, a sequence of optimal decisions is determined. Furthermore, the model provides an analytical solution to the minimal benefit the conflict has to generate. At the same time, this value marks the threshold that triggers the optimal outbreak of conflict. In the next step, the expected time of attack is determined and its sensitivity regarding various forms of variability during the non-violent period is discussed. Besides the provision of the optimal solution to the conflict decision, this contribution shows that uncertainty has an ambiguous effect. Similar to the third chapter, marginal variability in the non-violent period extends waiting even if the living conditions keep getting worse. Only the inclusion of non-marginal variability and the distinction between positive and negative variability provides a more realistic investment advice. In particular, if disasters are expected to occur more often without the outbreak of conflict, the rebel should attack earlier in order to enable a political, social and economic change. He or she should not suffer any longer because in average, the situation is not expected to improve without violence.

As the name of this doctoral thesis implies, the aim was to investigate the evaluation of large shocks of uncertain size occurring at an uncertain time. Furthermore, it discusses their effect on investment decisions and shows some applications. We can see that large stochastic shocks are a major component of investment decisions and investment behaviour in general. The most important conclusion that can be identified from this dissertation is the fact that different types of stochastic variability -namely marginal and non-marginal shocks- have opposed effects and every of them should be accounted for. While marginal variability does not matter for projects that are evaluated with the ENPV, large stochastic shocks, e.g. modelled by the Ito-Lévy Jump Diffusion process affect the project positively or negatively, depending on whether opportunities are more likely than threats. Many stochastic disasters can even make an initially beneficial project to become worthless. This ambiguous effect of variability is also obtained in the context of sequential investment decisions where the investor has the opportunity to postpone his or her investment decision to a later date. That is, marginal stochastic variability, as the volatility in the geometric Brownian motion, suggests to wait, collect more information and decide in a later period, while non-marginal stochastic variability may suggest to invest earlier. The reason for these results is that marginal stochastic shocks do not distinguish between beneficial and non-beneficial variability.

This doctoral thesis adds to the literature in that it provides a more general methodology that leads to more convincing investment rules: positive shocks (opportunities) have a different effect than negative shocks. Furthermore, it emphasizes the non-profitable consequences of neglecting either type of variability, especially of only considering marginal variability, such as modelled in the geometric Brownian motion. Hence, more general stochastic processes, such as the Ito-Lévy Jump Diffusion process, have to be implemented in economic decision making. The importance of this result becomes especially clear when non-marginal stochastic shocks such as disasters are considered. Large shocks, in contrast to marginal shocks, enter the expected value and therefore affect the decision. In other words, the investor obtains a discount parameter that is extended by a term stemming from the jump risk. With this thesis, a concept is provided that approaches the phenomenon which Knight calls uncertainty. That is, events which are not predictable in the sense of how severe they will be and at which point in time they will occur, are included into the analysis. For instance, we cannot state the probability of a certain disaster to happen during a specified time period. Furthermore, this modelling is able to provide analytical solutions to the total investment volume (if investment costs accumulate), the required minimum compensation that the investment needs to generate, and the expected optimal time of investment.

The consequences for particular investment projects are as follows: The example of the human capital investment decision shows that it may be better to stay in school for a while longer and gain even more benefits if waiting leads to more opportunities in the future career. If the investment project is instead about changing the status quo by investing in an action that prevents further disasters, then it may not be beneficial to wait and suffer any longer. An example of such a setting is given by the fourth chapter where investments in social conflicts are considered. Note that only looking at variances and volatilities would have prevented us from obtaining these insights.

The last chapter provides a general summary of the methodological progress that has been made in this doctoral thesis, as well as additional insights obtained by using the new methodology. It also gives an outlook for further research and application.

Chapter 2

Investment Under Threat of Disaster

This chapter is a joint work with Thomas Gries. It is a slightly revised version of the working paper No. 2014-04 that was published in the CIE Working Paper Series.

2.1 Introduction

Disasters[4] are an enormous hazard to economic activities and are defined as rare, devastating ecological, political, technical or economic events that occur at unpredictable points in time and with massive direct and threatening effects. For instance, shocks like the 9/11 terrorist attacks or political or even revolutionary riots such as the Arab Spring do not occur often, however, they have a severe impact on economic conditions. Similarly, in the economic dimension we have observed that events such as the Lehman Brothers bankruptcy in 2008 and the subsequent financial crisis are connected to the potential failure of large banks or even states. In the context of technical or natural disasters, Figure (2·1) suggests that on a global scale, the number and severity of disasters has increased since the 1970s. Hence, disasters were and will always be a significant phenomenon in economic reality.

In spite of their low probability, such major stochastic shocks cause uncertainty for the affected individuals. That is, in contrast to other marginal variability, disasters

[4] For a more detailed description and classification, see http://www.emdat.be/explanatory-notes.

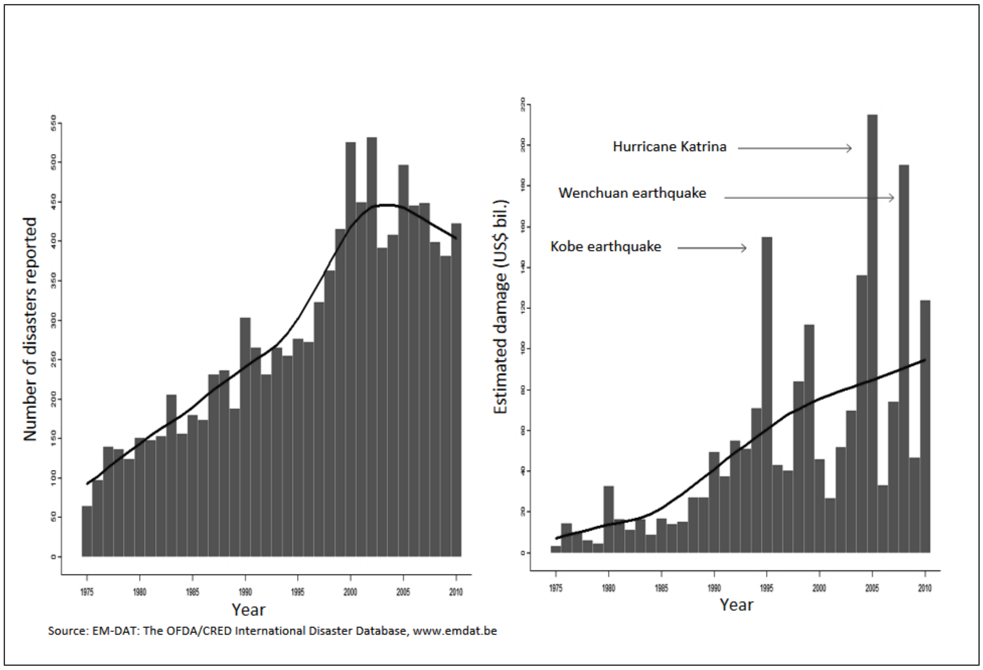

Figure 2·1: Natural Disasters reported in 1975-2010 in Numbers and caused Damage (US$ billion)

may not only strongly affect current conditions, they may even overthrow them. For this reason disasters are a central element of future developments and hence have to be included when evaluating projects. We show how the threat of disastrous events affects even simple evaluation methods, such as the Expected Net Present Value (ENPV). More precisely, we distinguish between marginal and non-marginal shocks by describing uncertain developments using more general stochastic processes (Ito-Lévy Jump Diffusion). For such a process we (i) confirm the standard result that the ENPV does not account for marginal variability. Furthermore, we (ii) obtain an additional element in the discount factor that summarizes the effects of non-marginal stochastic shocks on the expected project value. In other words, this parameter is an element of the discount factor in the evolution of the ENPV. Hence, while marginal variability has no effect on investment decision based on ENPV, non-marginal stochastic shocks do because, as we show, they may cause an apparently beneficial project to become worthless.

Hence, the questions we answer in this paper are: How can disasters be included into investment decisions based on ENPV? How does the potential occurrence of uncertain disasters affect the decision to realize an investment project? How can an investor cope with the uncertain timing and magnitude of the resulting effect in project evaluation and decision making?

During the last decade there have been attempts to include disasters into investment decisions, yet so far no consensus on how to do so appropriately has been found. Zeckhauser (1996) was one of the first to treat disasters as economic events and to define their costs as the sum of losses caused and the costs incurred by actions to reduce those losses. Although the costs of catastrophes have increased over time, reasons for neglecting disasters can still be found in the literature. Firstly, the comparatively low probability of disasters leads to a lack of comprehension so that, according to Kunreuther et al. (2001), individuals tend to ignore disasters and not to demand protection. Secondly, Kunreuther and Kleffner (1992) and Kunreuther (1996) find that agents often underestimate disaster probabilities and have high discount rates for future benefits. Finally, Kunreuther and Pauly (2004) argue that information costs for obtaining information about real probabilities are regarded as too high compared to the expected loss generated by disasters.

In the years that followed, all three arguments for neglecting disasters in decision making were discussed. In the context of natural disasters, Smith et al. (2006) emphasize that people indeed respond to natural disasters and that they do so in three different ways: they may self-protect, buy insurance, or move away from hazard prone areas. That is, households compare the costs and benefits of the three alternatives and make their choice by taking into account the harms caused by disasters, with the chosen reaction depending only on their income level. The first economic modelling of disasters and their inclusion into evaluation methods is provided by Sutter and Poitras (2010), who use the expected utility approach to show that people should account for disasters when deciding which type of house to build. They find that people substitute from manufactured homes in risk-prone areas. While both approaches – that of Smith et al. (2006) and Sutter and Poitras (2010) – are similar in that disasters matter to decision-making, neither addresses the problem that disasters may be underestimated by agents. In response, Lave and Apt (2006) claim that extreme natural events are more frequent than expected, and Viscusi (2009) shows that people do not appropriately evaluate risks from terrorism, natural disasters, and traffic accidents. He finds that death through terrorism is valued twice as high as death due to natural disaster but valued equally to death by accident. This paradoxical view of risks may explain decisions that are made in the context of natural disasters. Born and Viscusi (2006) find that insurance companies also suffer from major catastrophes and face higher losses after a disaster. As a consequence they raise insurance premiums in order to lower loss ratios in the following period. This ex-post adjustment indicates an inadequate evaluation of disasters in the past.

The next question that arises is, how disasters have been included into the evaluation of investment projects. Starting with the Cost Benefit Approach (CBA), we can see that while CBA under certainty is well developed[5] the issue of how to evaluate large uncertainty appropriately remains an unresolved puzzle. Graham (1981) was one of the first to extend CBA and to include uncertain outcomes with known probability. His simple expected utility approach makes it possible to determine an action with the best outcome and a value of the corresponding option. Although CBA is simple to apply, there are also major concerns that it is not capable of

[5] See, e.g., Prest and Turvey (1965), Sassone and Schaffer (1978), Layard and Glaister (1994).

adequately evaluating large uncertain events and disaster risk. For instance, Lave and Apt (2006) use CBA to determine the optimal size of a dam as flood protection and show that people refuse to buy insurance because of the above-mentioned undervaluation reasons.

Disaster risk evaluation, however, has been a major issue in financial economics, and more recently has also entered business cycle theory. Beginning with Merton (1975), jump processes are used to model rare events that cause non-marginal movements of values. He argues that most of the time an asset follows a Brownian motion and, with a known probability, jumps by random amplitude. Based on this argument, different conclusions about option values and business cycles[6] can be made for a set of simple jump processes. The occurrence of disasters was first modelled by simple downward jumps by Cox et al. (2000, 2004). As an extension, Yang and Zhang (2005) and Jang (2007) use a jump diffusion process to model the randomness of disasters, that is, their unknown frequency and impact. More general jump diffusion models are considered by Pham (1997), Kou (2002) and Kou and Wang (2003 a, b). Specifically, Pham (1997) and Kou (2002) generalize the Black and Scholes option pricing model in order to account for empirical phenomenons of asset prices such as volatility smiles and jump risks.[7] The recent generalization by Cai and Kou (2011) shows that a mixed-exponential jump diffusion model is able to approximate any other distribution as closely as possible. Furthermore, they prove that analytical solutions can be found for Laplace transforms of prices and sensitivity parameters

[6]More recently disasters have also been seen as important determinants of many other economic variables that drive business cycles. For instance, Gourio (2012) emphasizes that disasters such as Great Recessions depress employment, output, investment, stock prices, and interest rates, and increase expected returns on risky assets. This approach is applied to human capital theory in Bilkic et al. (2012).

[7]Kou and Wang (2003 a, b) also discuss various characteristics of jump processes, e.g. first passage times when introducing the double exponential distribution, and derive respective option prices.

of path-dependent options. Jump models are also used to describe random waiting periods between trades or hedging options. In this sense, Cartea and Meyer-Brandis (2010) utilize, among others, compound Poisson processes for waiting periods that are exponentially distributed and show an effect on option prices, while Alexander and Kaeck (2012) test different jump diffusion models and their ability to model hedging options. He shows that the inclusion of jumps is necessary to improve hedging performance. A further application of rare events in economic problems is shown in the discussion of the equity premium puzzle. Mehra and Prescott (1985) were the first to find high-risk premia in equities and thereby opened up a completely new research field. One explanation of this phenomenon is given by Rietz (1998) who claims that the possibility of an unlikely market crash is responsible for the additional price difference in equities. This explanation would not be taken up by the economic research community for almost 20 years. Then, Barro (2006) picked up Rietz's idea and found empirical evidence of economic disasters leading to higher risk premia. Since then this topic has been discussed in different variations. For instance, Barro and Ursua (2008) investigate in an empirical approach how wars can result in consumption disasters and thereby show a further application. As an extension of Rietz (1998), Gabaix (2008, 2012) discusses disasters as a determinant of ten macro-economic puzzles, such as the risk-free rate puzzle and excess volatility puzzle. Using calibration methods he shows that disasters can explain some of the economic phenomenons. A theoretical contribution in this context is provided by Wachter (2013) who uses jump diffusion processes to explain the equity premium puzzle. Hence, by deriving the premiums for equities under disaster risk, she shows that investors obtain large risk premiums when facing disaster risks compared to investors that do not.[8]

[8]This result generalizes the findings of Jarrow and Zhao (2006), where portfolio choices, when

The chapter is structured as follows. The first part of the next section discusses the most commonly used process for modelling investment decisions under uncertainty, namely the geometric Brownian motion, and shows its characteristics regarding the evaluation of uncertainties. The next part extends the geometric Brownian motion for disasters occurring at an uncertain point in time with an unpredictable magnitude and analyses the effects of the additional uncertainty component on the investment decision. In order to illustrate the effects, we use an example. The last section concludes.

2.2 Investment Project Evaluation Using Stochastic Processes

Investment project evaluation takes place in an uncertain future environment and hence requires a characterization of stochastic future outcomes. The simplest characterization of stochastic outcomes is a static probability distribution of values. However, for dynamic evaluation methods such as the ENPV technique, investors look at a full sequence of outcomes in each future period and try to come as close as possible to a characterization of this sequence of periods. Hence, they would try to characterize the full time path of stochastic realizations as well as possible. So far, in continuous time, stochastic processes are the only broadly understood form of such stochastic time paths. Therefore, applying stochastic processes, usually a geometric Brownian motion, seems the most appropriate way of approaching this objective and including the time dimension of future events.

Hence, as a first benchmark, we introduce the geometric Brownian motion and show how this stochastic process enters the ENPV. In the next step we generalize this simple stochastic process to evaluate uncertain events such as disasters occurring

downside loss-averse, depend on the presence of disasters.

randomly and with an uncertain magnitude. Finally, we use an example to show that going beyond marginal shocks and introducing stochastic non-marginal shocks affects investment decisions even for this simplest evaluation method.

2.2.1 Evaluation with Geometric Brownian Motion

Since Black and Scholes (1973), the geometric Brownian motion is one of the most frequently used stochastic processes for modelling values of derivatives and investment projects. The evolution of benefits $\tilde{P}(t) \in \mathbb{R}$ can be characterized by a stochastic differential equation (SDE)

$$d\tilde{P} = \tilde{\alpha}_p \tilde{P} dt + \tilde{\sigma}_p \tilde{P} d\tilde{W}_p, \text{ with } \tilde{P}(0) = \tilde{p}_0, \tag{2.1}$$

where $d\tilde{W}_p$ is the increment of the standard Wiener Process. Equation (2.1) defines a stochastic process with continuous trajectories (see Figure 2·2) that have a trend $\tilde{\alpha}_p \in \mathbb{R}$ and marginal fluctuations depicted by the volatility $\tilde{\sigma}_p \in \mathbb{R}_+$.

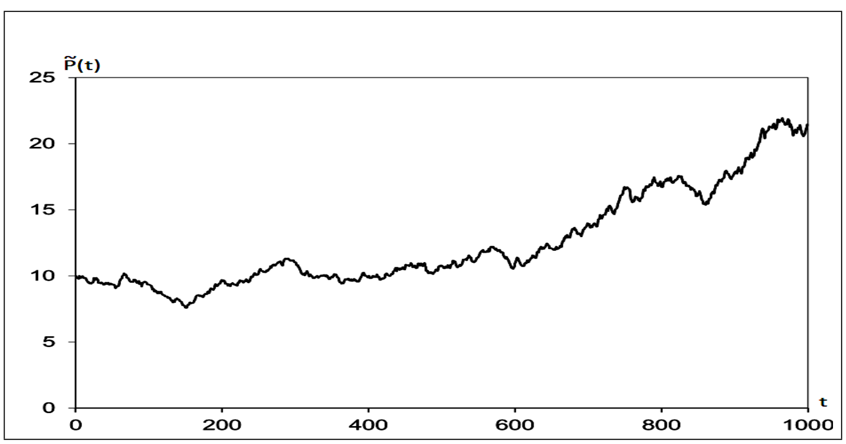

Figure 2·2: Path of the Geometric Brownian Motion

For $\tilde{\alpha}_p > 0$ the project benefits increase on average and have the expected value[9] of

$$E\tilde{P}(t) = \tilde{P}(0)e^{\tilde{\alpha}_p t}.$$

Furthermore, the larger the constant volatility, $\tilde{\sigma}_p$, the more the project value fluctuates around its expected value $E\tilde{P}(t)$, regardless whether this deviation is positive or negative. Note that for the geometric Brownian motion fluctuations are modeled continuously so that only marginal differences between one point in time and another are described. With $\tilde{\sigma}_p = 0$ (2.1) simplifies to an ordinary differential equation with solution $\tilde{P}(t) = \tilde{P}(0)e^{\tilde{\alpha}_p t}$.

Now suppose that an agent considers investing in an investment project with a value that evolves according to (2.1) and has costs $I_p \in \mathbb{R}_+$. To evaluate this project with the ENPV technique he or she has to accumulate the discounted benefits per period and reduce this by the project costs.

Proposition 1 *Let $\tilde{P}$ be defined as in (2.1) and $T \in \mathbb{R}_+$ being the time of investment. Then the Expected Net Present Value ($ENPV_1$) of future benefits described by $\tilde{P}$ is*

$$ENPV_1 = -I_p + E\left(\int_T^\infty \tilde{P}e^{-r(t-T)}dt\right) = -I_p + \frac{\tilde{P}(T)}{r - \tilde{\alpha}_p}; \quad r > \tilde{\alpha}_p, \tag{2.2}$$

with r being the risk-free interest rate and I_p being investment costs.

Proof. See Appendix A.1. ■

According to (2.2), the ENPV consists of the project value at time T discounted by the difference between the risk-free interest rate r and the drift $\tilde{\alpha}_p$, and reduced

[9] For a detailed derivation of the expected value, see Dixit and Pindyck (1994).

by the investment costs I_p. Hence, the investor will carry out the project only if $ENPV_1 \geq 0$. The decision does not depend on the volatility $\tilde{\sigma}_p$, so that marginal shocks have no effect on the investment decision. In other words, using the ENPV technique and taking the geometric Brownian motion for describing the stochastic income path of an investment project, does not account for any shocks, not even for marginal shocks from one moment to the next. This is consistent with the interpretation of the ENPV method as a risk-neglecting approach. Although Black and Scholes' approach was a benchmark in the evolution of evaluation theories, Merton (1975) was the first to criticize this drawback of the geometric Brownian motion. He points out that prices do not move according to the geometric Brownian motion, nor does trade take place continuously. He argues that stock prices can never be represented by continuous stochastic processes because uncertainty produced by incoming important news can lead to an immediate non-marginal upward or downward movement in prices that may affect investment decisions.

2.2.2 Evaluation with Jump Processes

To model disasters as large stochastic events that are non-marginal stochastic shocks in the benefit stream, we introduce the discontinuous counterpart of the geometric Brownian motion. Specifically, this is a stochastic process that is constructed by continuous and discontinuous Lévy processes.[10] In this chapter, we follow the approach of Pham (1997) and Kou and Wang (2003a) and combine a jump process with the geometric Brownian motion. The resulting geometric Ito-Lévy Jump Dif-

[10] A more detailed description of Lévy processes, especially of the prerequisites for definitions, can be found in, e.g., Cont and Eberlein (2010), Oksendal and Sulem (2007), Andersen et al. (2009) or Protter (1990).

fusion process $P \in \mathbb{R}$ is defined by the stochastic differential equation

$$dP = P\alpha_p dt + P\sigma_p dW_p + P\int\limits_{U_p} z_p N_p(t, dz_p), \quad P(0) = p_0, \tag{2.3}$$

with α_p, $\sigma_p \in \mathbb{R}_+$ constant, and dW_p denoting the increment of the Wiener process. N_p describes a Poisson process with intensity λ_p and the integral $\int\limits_{U_p} z_p N_p(t, dz_p)$ itself describes a compound Poisson process. The intensity of the compound Poisson process is given by the Lévy measure $\upsilon_p(dz_p)dt = \lambda_p h_p(dz_p)dt$, where h_p is the distribution of the jump step heights. Furthermore, $\int\limits_{U_p} z_p N_p(t, dz_p)$ can be interpreted as a stochastic process that models positive or negative reactions of benefits to offered opportunities and threats. The direction and magnitude of one jump is represented by the step height $\triangle P(t) := z_p = P(t) - P(t^-) \in U_p$, with $U_p \subseteq (-1, 0)$ being a Borel set and $P_p(t^-)$ denoting the left limit of P_p in t.[11] According to the representation of the stochastic process in (2.3), the project value evolves like a geometric Brownian motion and jumps upwards or downwards at random points in time. Figure 2·3 is an example of a path of a geometric Ito-Lévy Jump Diffusion process.

The Ito-Lévy Jump Diffusion process has two variability components, σ_p for marginal shocks and $\int\limits_{U_p} z_p N_p(t, dz_p)$ for non-marginal stochastic shocks. Non-marginal shocks are characterized most generally. In particular, with this model non-marginal jumps may happen at an uncertain time and have a large but uncertain magnitude. That is, we do not know when and with what impact these large shocks occur. However, we may have information about the intensity λ_p and the distribution of jump heights h_p.

With this combination of continuous volatility and a jump process, we can now

[11] We have to assume U_p having a lower boundary not lower than -1, because, as will be seen later, the solution to the differential equation dP has a solution only for $z_p > -1$.

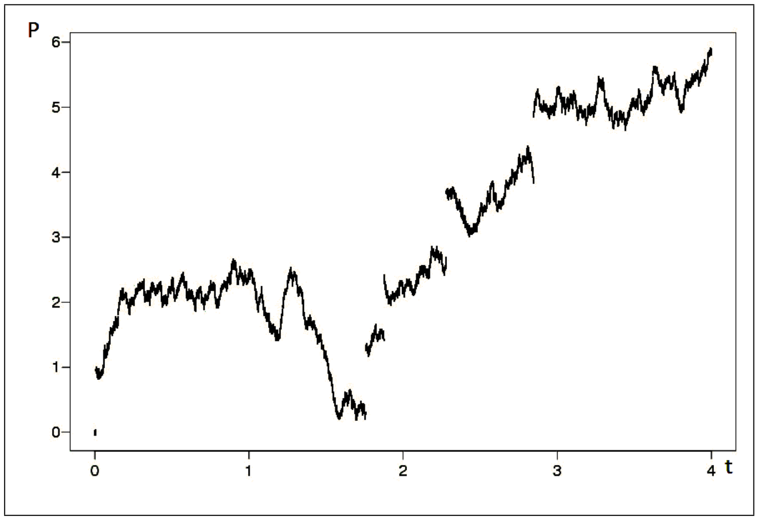

Figure 2·3: Path of a Geometric Ito-Lévy Jump Diffusion Process

describe the complete random path of income where marginal and non-marginal shocks in the benefit stream are present. As Knight (1921) argues, when future outcomes are unknown, they can either be risky or uncertain. While for Knight, risk implies that probability distributions can be stated, uncertainty describes a situation where no such specification can be made. With this modelling we try to move one step towards Knight's notion of uncertainty. We implement more complex stochastics in this model, and hence introduce a degree of uncertainty and randomness that cannot be covered by a simple probability distribution. We have no information on the likelihood and timing of the next shock and the severity with which the next shock will strike. That is, with this kind of stochastic modelling future developments are so uncertain that it is impossible to know the probability that a disaster of size x will occur at time t. Moreover, we cannot even state the probability of a certain disaster happening during a specified time period from now on. In other words, if an engineer states that the probability of a just finished dam collapsing during the next 100 years is 1×10^{-100} and hence negligible, he still uses a probabilistic model of risk.

In contrast, using more complex stochastics like Ito-Lévy Jump Diffusion processes, this kind of statement would be impossible since we cannot even give a probability of this disaster happening during the next 100 years. As a result, simple probabilistic statements, in particular when considering large, sometimes even "overthrowing" events, are a misleading and insufficient description of the degree of randomness under these circumstances.

As we illustrate, we can still evaluate this rather uncertain future development using the simple ENPV technique. Again, we consider an agent who takes a decision on a project with a benefit stream P. In order to see how the two risk measures affect the investment decision, we can compute the ENPV of the project. For this we first need the expected value of the process.

Proposition 2 *Let P be defined as in (2.3), f_p^{-1} denoting the inverse function of $f_p(z_p) = \ln(1+z_p)$ and $u_p \in \mathbb{C}$ constant. Assume that the condition $\int_{U_p} e^{u_p z_p} \upsilon(dz_p) < \infty$ of finite exponential moments holds, so that the moments of the stochastic process in (2.3) are also finite. Then the expected value of P is determined by*

$$
\begin{aligned}
EP(t) &= P(0)\exp\left[t\left(\alpha_p + \int\limits_{f_p^{-1}(U_p)} z_p \upsilon_p(dz_p) + \int\limits_{U_p} \ln(1+z_p) - z_p \upsilon_p(dz_p)\right)\right] \qquad (2.4)\\
&= P(0)\exp\left[t\left(\alpha_p + \lambda_p \int\limits_{f_p^{-1}(U_p)} z_p h_p(dz_p) + \lambda_p \int\limits_{U_p} \ln(1+z_p) - z_p h_p(dz_p)\right)\right].
\end{aligned}
$$

Proof. See Appendix A.2. ■

In contrast to the geometric Brownian motion, the expected value in (2.4) does not only depend on the drift but also on the direction and magnitude given by the disasters in the benefit stream. In particular, (2.4) is not automatically an increasing function for $\alpha_p > 0$ since the average growth rate (also known as the overall drift) is determined by $\int\limits_{f_p^{-1}(U_p)} z_p \upsilon_p(dz_p) + \int\limits_{U_p} \ln(1+z_p) - z_p \upsilon_p(dz_p)$, which can either be positive or negative depending on whether more upward or more downward jumps occur. Hence, if we consider disasters where only negative jumps occur, $\int\limits_{f_p^{-1}(U_p)} z_p \upsilon_p(dz_p) + \int\limits_{U_p} \ln(1+z_p) - z_p \upsilon_p(dz_p)$ will be negative and we obtain a function P that increases according to $\alpha_p > 0$ and jumps downward due to threatening events. In order to simplify the notion we replace $\int\limits_{f_p^{-1}(U_p)} z_p h_p(dz_p) + \int\limits_{U_p} \ln(1+z_p) - z_p h_p(dz_p)$ by a disaster variable $\delta < 0$ from now on and write $EP(t) = P(0) \exp\left[t\left(\alpha_p + \lambda_p \delta\right)\right]$.

In the next step we compute the ENPV for a project that evolves according to a geometric Ito-Lévy Jump Diffusion process.

Proposition 3 *Let P be defined as in (2.3). Then the Expected Net Present Value of future benefits described by P is*

$$\begin{aligned} ENPV_2 &= -I_p + E\int\limits_T^\infty e^{-r(t-T)} P(t)dt \qquad (2.5) \\ &= -I_p + \frac{P(T)}{(r - \lambda_p \delta - \alpha_p)}, \end{aligned}$$

for $r > \lambda_p \delta + \alpha_p$.

Proof. See Appendix A.2. ■

From (2.5) we can see that the disaster-facing investor evaluates disasters only by looking at the Expected Net Present Value of the project. $ENPV_2$ depends on the benefit value in T and the sum of the integrals $\lambda_p \delta$. Since $\lambda_p \delta$ itself depends on the intensity and step height of the jumps, more disasters mean more threats for the investor, leading to a negative value of both integrals. In this case negative jumps act as an additional element of the discount parameter that in turn decreases the ENPV. As a result, the existence of disastrous threats, the frequency and extent to which they may occur, determine the evaluation of the expected project value. Marginal shocks, in contrast, indicated by volatility σ_p, do not affect the ENPV evaluation framework.

2.2.3 Effects of Disasters on Expected Net Present Value

Having determined the ENPV of a project, we see that large uncertain shocks matter, whereas volatility does not. In this section we look at the derivatives to analyse how these large shocks affect the investment decision.

By using the ENPV method, which allows for discontinuous project earnings described by Ito-Lévy Jump Diffusions, we are able to determine the effect of an increasing number of disasters and of an increasing volatility on investment projects. Hence, the derivatives of the ENPV with respect to the jump intensity λ_p and to volatility σ_p are[12]

$$\frac{\partial ENPV_2}{\partial \lambda_p} = -\frac{\delta P(T)}{(r - \lambda_p \delta - \alpha_p)^2} < 0, \qquad \frac{\partial ENPV_2}{\partial \sigma_p} = 0.$$

An increasing frequency of disasters will lead to a positive numerator, making the derivative of the Expected Net Present Value negative. Specifically, if the investment project is prone to more disasters so that disasters occur more often, the ENPV will decrease and the project will lose in value. In other words, major damage caused by natural, technological, or other disasters can happen so frequently that the real investment project may not pay off any longer. In contrast, small variations in the project value have no effect on the investment decision of the investor. As a consequence, an investor would overestimate the project value if he or she insufficiently accounted for the possibility occurrence of disasters. He would not see the

[12] For a proof see Appendix A.3.

potential loss that is connected to large uncertain shocks and would instead invest in a potentially worthless project.

In order to show the effect of an increasing number of disasters, we provide a graphical illustration in addition to our general analytical findings. We assume that the jump sizes z_p have the double exponential distribution similar to Kou and Wang (2003b)[13]

$$h(z) = m_p \eta_{p_1} e^{-\eta_{p_1} z_p} 1_{\{z_p \geq 0\}} + n_p \eta_{p_2} e^{\eta_{p_2} z_p} 1_{\{z_p < 0\}},$$

where m_p is the probability of a positive jump and n_p of a negative jump, respectively, with $m_p + n_p = 1$. $\frac{1}{\eta_{p_1}}$ and $\frac{1}{\eta_{p_2}}$ denote the means of the two exponential distributions. Each exponential distribution can be interpreted as a distribution of the waiting period until a positive or a negative jump occurs. In other words, during this waiting period the occurrence of fundamental opportunities and disasters affects the decision to invest.

To illustrate,[14] we assume the investment cost I_p to be equal to 1, the risk free interest rate r equal to 5% and the average growth rate α_p of benefits equal to 2%. Furthermore, the probability of a downward jump is assumed to be 100% compared to a 0% probability of upward jumps as we want to analyse the effect of disasters. In a period we assume that at $\frac{1}{\eta_{p_1}} = \infty$, the mean waiting period for an opportunity is four times longer than for a threat with $\frac{1}{\eta_{p_2}} = \frac{1}{2}$. Then we double the frequency and impact of negative jumps stepwise while beginning with a 5% impact of a downward jump and a mean arrival rate of 10%. Figure 2·4 shows that more negative jumps with a higher devastating impact rapidly decrease the ENPV, compared to the dashed line where no jumps occur. For instance, doubling z_p and λ_p from 10% (in point A)

[13] A possible generalization is to use a mixed exponential distribution, provided in Cai and Kou (2011).

[14] For the detailed computation results, see Table A.1 in Appendix A.3.

to 20% (in point B) decreases the ENPV by 32% and then to a point when the project value even turns negative (in point C). In this case, the threatening impact of disasters on real investment projects outweighs their benefits so that in some cases they should not be carried out.

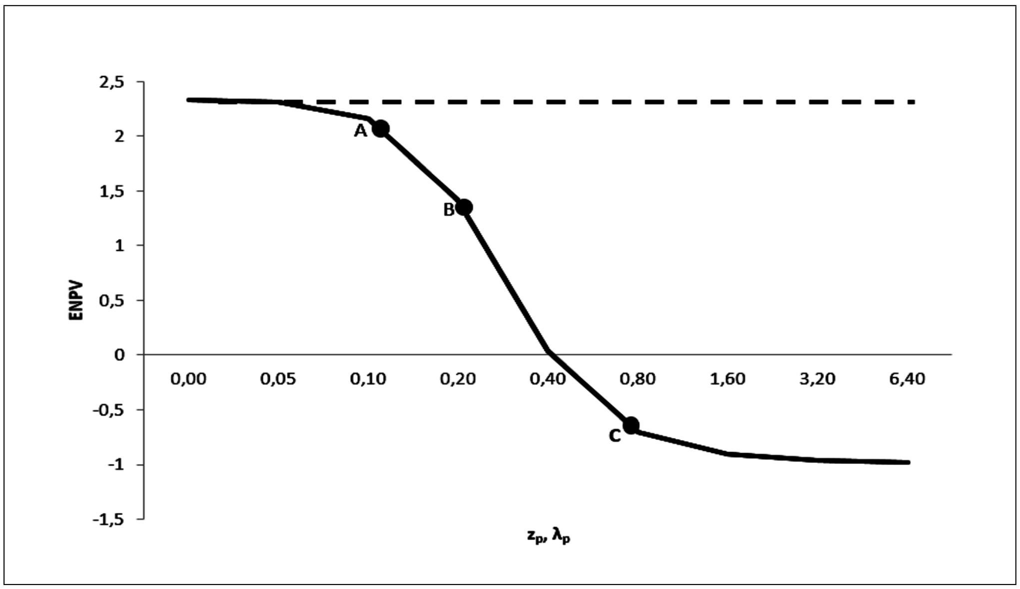

Figure 2·4: Effects of an Increasing Frequency and Magnitude of Disasters

As a general result, we obtain rules for how changes in these parameters translate into project values in the ENPV approach. Fortunately, these rules are rather simple, appearing almost like rules of thumb, given that the occurrence and impact of disasters can be described by systematic parameters such as frequency of occurrence and extent of damage.

2.3 Conclusion

This paper addresses the impact of disasters on the value of investment projects. As the number of disasters has significantly increased during the last decades, their impact is a growing threat to long term investment projects. Disasters are large events with a highly uncertain occurrence and impact. Within the most commonly used methods of project evaluation there is no framework for evaluating effects of disasters. This paper provides a theoretical concept for evaluating disastrous uncertain shocks using the simple ENPV approach. We argue that simple probabilistic statements, in particular when considering large or even overthrowing events, are misleading and an insufficient description of the degree of randomness. Hence, we implement a more complex stochastics using a stochastic Ito-Lévy Jump Diffusion process. With such a stochastic process we do not need to state the probability of a certain disaster occurring during a specified future time period. However, we can still evaluate this rather uncertain future using the simple ENPV technique. We show that in contrast to the well-known fact that marginal shocks, indicated by volatility, do not affect the ENPV, large stochastic shocks do. Parameters characterizing the frequency and the size of these large shocks are elements of the discount factor. We show that a higher frequency and impact of disasters rapidly decreases the value of a project even to negative values. Disregarding potential disasters leads to an overestimation of projects.

Chapter 3

Stay in School or Start Working? - The Human Capital Investment Decision Under Uncertainty and Irreversibility

This chapter is a joint work with Thomas Gries and Margarethe Pilichowski. A former version of it was published in 2012 in Labour Economics 19 (5), pp. 706-717.

3.1 Introduction

After the first contribution of this doctoral thesis has shown the effect of a large stochastic variability on investment decisions, the next contribution uses the previously developed ENPV for a sequential investment. That is, in a two period model, the first phase is characterized by accumulating schooling costs and the second by a highly uncertain earning stream. The investor makes a choice about when to end the first and to begin the second by comparing the continuation value with the potential ENPV of an immediate change and the decision is modelled by the real option approach. The model determines the required compensation that the investment needs to generate as well as the expected optimal time to make the change. The developed model is put in the context of the human capital investment decision where a student decides about the optimal labour market entry.

Education is obtained during a long process of the accumulation of knowledge and abilities. Hence, formal schooling is a learning and investment process that often lasts into one's mid-twenties. When a young person makes plans for the future one of the biggest problems is uncertainty.[15] The success of a long education is as uncertain as the process of earning income during a long working life. As time goes on, students repeatedly consider whether to continue their education or enter the labour market. During this sequential process of decision making each moment's conditions determine the eventual attainment level.

Recent literature shows that real option theory can be applied to take into account uncertain time processes and irreversibility in schooling and human capital accumulation decisions. While Weisbrod (1962) and more formally Comay et al. (1973) suggested this way of thinking more than 30 years ago, a transfer of formal option theory - as established by Dixit and Pindyck (1994) - was suggested only recently. Hogan and Walker (2007) apply real option theory to human capital decisions. In their model, at any time a student has the option to leave school to work for a wage that reflects the years spent in school. The decision to leave school is irreversible, so once the student has finished education he or she cannot return. They conclude that high returns on education and increasing risk will cause students to stay in school longer. They also analyse how progressive taxation and education subsidies affect schooling decisions and show that progressive taxes tend to reduce educational attainment. Jacobs (2007) uses the real option approach as well. Unlike Hogan and

[15] The first analysis of investment in human capital under uncertainty was conducted by Levhari and Weiss (1974). Later, e.g. Eaton and Rosen (1980) extended this framework. Williams (1978) examined risky investments in education using a two-period, mean-variance portfolio model. Groot and Oosterbeek (1992) discussed the effects of uncertain future earnings and the probability of unemployment on the duration of schooling, considering several sources of risk, and Hanchane et al. (2006) developed a continuous time dynamic programming model which accounts for several sources of uncertainty with regard to earnings and labor market conditions. They showed that the global effect of uncertainty is negative, except when a sufficiently high risk premium exists.

Walker (2007), he uses a discrete time approach and states that the decision to start learning is irreversible. The option value stems from the fact that an individual could wait to enrol and would only do so once the returns are sufficiently large to compensate for the lost option value. The sunk cost of the investment consists of tuition costs and foregone labour earnings.

More recent empirical literature on human capital investments suggests that the functional form of the Mincer model (1974) no longer adequately describes labour earnings for U.S. workers.[16] Heckman et al. (2003, 2006) test and reject the assumptions for using the Mincer model to estimate the internal rate of return. Heckman et al. (2008) emphasize that estimates should account for non-linearity and non-separabilities in earnings functions, income taxes, and tuition. In line with these findings is the idea of introducing risk and other non-pecuniary elements into the empirical model.[17] In addition, Heckman, Lochner, and Todd (2006) explain why option values should be included in the decision, and show how option values invalidate the internal rate of return as an investment choice criterion. "Our analysis points to a need for more empirical studies that incorporate the sequential nature of individual schooling decisions and uncertainty about education costs and future earnings to help determine their importance. We report evidence on estimated option values from the recent empirical literature using rich panel data sources that enable analysts to answer questions that could not be answered with the cross section data available to Mincer in the 1960s." [Heckman et al., 2006, p. 6]. All these findings encourage a closer look at the impact of real option theory on human capital investment decision under uncertainty and generate a more comprehensive theoretical framework.

[16] This development starts, e.g. with Katz and Autor (1999).

[17] See, e.g., Cunha et al. (2005), Carneiro et al. (2003), Belzil and Leonardi (2007a, b) or Hartog et al. (2007).

Departing from the model suggested by Hogan and Walker (2007), we discuss how uncertain time processes determine the duration of schooling and - with the timing decision to leave school - the accumulation of human capital. We extend their framework by 1) adding accumulated education costs during schooling, 2) considering complete earnings profiles including entry-level wage, sheepskin effects and earning dynamics, and 3) discussing the option value of schooling introducing potential career opportunities or threats of unemployment modeled as major uncertain events connected with particular education achievements.
In order to discuss these problems we proceed as follows. In section 3.2, we introduce the real option base model to determine the expected time of leaving school for a continuous process of schooling. In section 3.3, we solve the base model, and discuss comparative statics. In section 3.4 we extend the model by introducing different levels of formal qualification and discuss the implications for option values with respect to sheepskin effects and major random events connected to particular formal education achievements. In section 3.5 we conclude.

3.2 Basic Model

The level of education a student attains is a result of a dynamic sequential decision process. Even if an immediate labour market entry may have some benefits, it is possible that staying in school is the better option. To model this uncertain investment and timing decision problem, Hogan and Walker (2007) suggest the real option approach in terms of a dynamic programming model from which we depart. In our model the sequential timing decision has three elements: 1) accumulated investment costs of schooling, 2) the earning profiles, starting with the entry-level wage when working life begins and developing as a dynamic income stream, and 3) the value of postponing the working life through longer education to potentially achieve a better income track, or the value of not to tie oneself to a specific uncertain earnings stream. In section 3.4, we discuss that the option value of education can account for sheepskin effects, potential large opportunities or threats, and remaining flexible. As this decision is repeated, we look at a multi-stage sequence of decisions that add up to the entire duration of schooling and the eventual level of academic achievement.

Investment Costs of Schooling In this model, the individual costs of a successfully completed year of schooling are defined by $C_h \in \mathbb{R}$ and, from today's perspective ($t = 0$), accumulate each year until the end of the student's education.[18] The total investment expenditure $I_h(T)$ is dynamic and increases with each year of schooling. Hence, at time $T \in \mathbb{R}_+$, the end of formal education, the current value of total schooling costs[19] is

$$I_h(T) = \int_0^T C_h e^{r(T-t)} dt + \bar{C}_h, \tag{3.1}$$

where r is the risk-free interest rate and $\bar{C}_h$ are the given costs of successfully graduating, finding adequate employment and entering the market.

Earnings Profile Education not only generates costs, but also provides access to different earnings profiles. On the one hand, schooling generates a differential in the entry-level wage when entering the labour market; on the other, it may lead to a change in the dynamics and risk of the income stream during working life.

[18] Recent empirical studies suggest that education costs are an important ingredient of the education decision (see, e.g., Heckman, 2008). By including the annual accumulative cost of schooling, we depart from Hogan and Walker (2007), who do not consider education costs.

[19] Education costs per period C_h could include accounting for in school utility. Hence C_h represent "general costs" per period so that the willingness to pay for the "utility of schooling" could be substracted and the costs could be regarded as net costs including utility benefits.

Entry-level wage: The initial level of the income path, namely the entry-level wage, is the first element of the earning profile. As it is linked to educational achievement, an additional (successful) year of schooling leads to a higher entry-level wage and therefore to an increased level of the earnings stream.[20] Many random elements determine the wage when entering the labour market. Hence, we describe the development of entry-level wages during formal education as a Brownian motion $\tilde{Y}(t) \in \mathbb{R}$

$$d\tilde{Y} = \tilde{\alpha}_h \tilde{Y} dt + \tilde{\sigma}_h \tilde{Y} d\tilde{W}_h \quad \text{with } \tilde{Y}(0) = \tilde{y}_0, \quad 0 < t < T, \tag{3.2}$$

where $\tilde{\sigma}_h \in \mathbb{R}_+$ and $d\tilde{W}_h$ denote a constant volatility and the increments of a standard Wiener process, respectively. $\tilde{\alpha}_h \in \mathbb{R}_+$ is the expected marginal differential in income level with respect to marginal schooling time and educational improvement (expected rate of market reward). This change in the level of the income path is part of the total income reward generated by the schooling process.

Dynamics and value of the income stream: The second element of the earnings profile is the dynamic development of the lifetime earnings stream. Because stylized facts indicate that it is linked to educational attainment, we use a random process with a trend and random elements. In general, individual income dynamics are driven by a stochastic earning process described by the geometric Brownian motion $Y \in \mathbb{R}$

$$dY = \alpha_h Y dt + \sigma_h Y dW_h \quad \text{with } Y(0) = y_0, \quad T < t, \tag{3.3}$$

with dW_h denoting the increments of a standard Wiener process. Upon entering the

[20] Wang and Bai (2003) examine how variations in uncertainty in labor productivity affect specific human capital investment and wage. They find a positive correlation between wage and specific human capital.

market ($t > T$), the student faces a stochastic revenue stream which is characterized by an expected average growth rate $\alpha_h \in \mathbb{R}_+$ and elements of uncertainty depicted by a constant volatility $\sigma_h \in \mathbb{R}_+$. To simplify the model, we assume α_h to be constant, whereas in the real world an earnings profile would not be linear but would decrease at the end of one's working life.[21] Once working life begins, the earnings stream follows a random process and other opportunities are ruled out. Hence, the future income stream will specify the economic value of the achieved level of education. For a risk neutral individual, the gross value of human capital (education wealth) V_h^{gross} is given by the expected present value of the earnings stream $\{Y_h(t)\}$. For simplicity, the individual has an infinite lifespan. To determine the ENPV of human capital (net wealth of education), the expected gross value (3.4) has to be adjusted for individual education costs $I_h(T)$ accumulated during the time of schooling (3.1)[22]

$$V_h^{gross} = E\left(\int_T^{\infty} Y_h e^{-r(t-T)} dt\right) = \frac{Y_h(T)}{r - \alpha_h}; \quad r > \alpha_h. \tag{3.4}$$

$$V_h = V_h^{gross} - I_h(T).$$

Option Value of Waiting Apart from affecting the earning profile, the duration of education corresponds to the value of the option of deferring market entry so as not to be tied to a lifetime earnings profile with the corresponding risk and irreversibility. Dixit (1989) as well as Dixit and Pindyck (1994) demonstrate that waiting has a value in the context of a firm's investment decision. In line with this approach, further

[21] For simplicity, we stay with a geometric Brownian motion even though we could replace it by an arithmetic Ornstein-Uhlenbeck process, as also suggested by Hogan and Walker (2007), and derive similar results plus an additional constant. At this point, we also assume identical a-levels no matter how many years of schooling were completed, and we do not distinguish between different formal educational levels such as primary, secondary or tertiary education. Later in this paper, we distinguish between different earnings profiles determined by the attained formal qualification including sheepskin effects.

[22] See Appendix B.2.1.

education could open up additional information and unforeseen opportunities and therefore lead to a higher earnings profile. In reality, the decision to complete one's education with a certain degree is surely not as strict as suggested by the expression "irreversible". Returning to the education system can be modeled by exit options, which we leave for a future extension to the present model. According to Dixit and Pindyck (1994), for the option value F_h for the Brownian motion (3.2), the Hamilton-Jacobi-Bellman (H-J-B) equation holds[23]

$$rF_h dt = E(dF_h).$$

Decision Problem The education decision is a timing problem concerning the entry into working life. We need to compare the net value of education V_h (for any educational achievement) with the option value F_h of further education and a better income profile. Once the net value of education and the option value of waiting have been determined, the question of whether or not to wait for another period will be answered by the solution to

$$\max \left\{ V_h(T), F_h(T) \right\}.$$

The student will decide in favour of another year of school if the option value of waiting is higher than the ENPV of the earning stream. Solving this continuous decision problem determines the time of entry into the labour market.

[23] See Appendix B.2.2.

3.3 Solving for Expected Time of Leaving School

The expected time of entering the labour market can be obtained in three steps. First, we determine the threshold $Y^*(T)$, which represents the entry-level wage required to make one's education profitable. Beyond the threshold, the value of the earning stream becomes higher than the option value of waiting and the student enters the labour market. Second, the student simultaneously observes the development of the relevant entry-level wage $\tilde{Y}(T)$ in the market, compares the threshold for his or her academic attainment with the corresponding current entry-level wage, and verifies if the threshold has already been reached. Third, if he or she decides to stay at school he or she will predict the expected duration of schooling.

Entry Threshold In order to determine the income value that triggers the switch, we need to consider the standard conditions of a stochastic dynamic programming problem. In addition to the H-J-B equation for the option value F_h and applying Ito's Lemma to dF_h, we have to use the well-known boundary conditions, namely (3.5), the value matching condition (3.6), and the smooth pasting condition (3.7)

$$F_h(0) = 0, \tag{3.5}$$

$$F_h(Y^*) = V_h^{gross}(Y^*) - I_h(T), \tag{3.6}$$

$$\frac{dF_h(Y^*)}{dY} = \frac{d(V_h^{gross}(Y^*) - I_h(T))}{dY} \tag{3.7}$$

to solve for the income threshold Y^*. Once this threshold is reached the student decides to enter the labour market.

Proposition 4 *For an accumulation of constant costs per year of successful schooling* (3.1)*, a sequence of increasing earning levels through schooling described by* (3.2)*, and an earning dynamics after market entry following* (3.3) *the threshold* $Y^*(T)$ *that* *triggers the start of the earning/working process is*

$$
\begin{aligned}
Y^*(T) &= \frac{\beta_h}{\beta_h - 1}(r - \alpha_h)\left[\frac{C_h}{r}\left(e^{rT} - 1\right) + \bar{C}_h\right] \qquad (3.8) \\
&= \frac{\beta_h}{\beta_h - 1}(r - \alpha_h) I_h(T), \\
\textit{with} \quad \beta_h &= \frac{1}{2} - \frac{\tilde{\alpha}_h}{\tilde{\sigma}_h^2} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\alpha}_h}{\tilde{\sigma}_h^2}\right)^2 + \frac{2r}{\tilde{\sigma}_h^2}}, \qquad (3.9) \\
\textit{and} \quad r &> \alpha_h.
\end{aligned}
$$

Proof. For a proof see Appendix B.2.3. ■

Each additional year of schooling dynamically adds to the total costs of education, so the investment costs increase over time. The threshold changes with the duration of schooling, i.e. it is a continuous function of time. A student would only complete an additional year if he or she is rewarded by a higher entry-level income.

Expected First-Time Realization of Entry-Level Wages If the random entry-level wage observable in the market matches the threshold, the education process terminates. The point in time when we expect this match for the first time, is called the "first passage time", which is determined analytically in the next section.

We first consider an instrument that we will call the expected first-time realization of entry-level wages. For the random process $\tilde{Y}$ (see (3.2)) we derive the expected time of first realizing a certain entry-level wage $\tilde{y}_i \in [\tilde{y}_0, \infty)$ (given today's value $\tilde{y}_0$). By using the Girsanov theorem we determine the probability density function[24] of $\tilde{T}_i$ which is sometimes referred to as the Inverse Gaussian Distribution.[25] Hence, we

[24] An extensive discussion is offered by Karatzas and Shreve (1991, p. 196) and Karlin and Taylor (1975, p. 363).

[25] The term "inverse Gaussian distribution" stems from the inverse relationship between the cumulative generating functions of these distributions and those of the Gaussian distributions. For a detailed discussion of the inverse Gaussian distribution see Johnson et al. Dixit (1993).

can write the expected first-time realization as a function of $\tilde{y}_i/\tilde{y}_0$

Proposition 5 *For the Brownian motion (3.2), the expected time of first realizing* $E\tilde{T}_h$ *of each entry-level wage* $\tilde{y} \in [\tilde{y}_0, \infty)$ *is a function of* $\tilde{y}/\tilde{y}_0$*. Hence, the expected time until any entry-level wage* $\tilde{y}$ *is reached for the first time is*

$$E\tilde{T}_h = \frac{1}{\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2} \ln\left(\frac{\tilde{y}}{\tilde{y}_0}\right). \tag{3.10}$$

Proof. For a proof see Appendix B.3. ■

This expected time of first realizing $E\tilde{T}_h$ for all values of entry-level wages can be drawn as the $E\tilde{T}_h$ curve in Figure 3·1.

Expected Time of Leaving School When planning his or her career, a student has to simultaneously observe the entry-level wages and the income threshold that will trigger market entry. In order to find the first passage time we use two types of information available. First, the student knows the threshold $Y^*(T)$ that triggers market entry for each duration of schooling T. Second, from the properties of the Brownian motion (3.2) the expected first-time realization of all initial income values, that is, the time when a certain entry-level wage $\tilde{Y}$ is expected to be reached for the first time $E\tilde{T}_h$, is known as well.

Hence, if the entry-level wage $\tilde{y}$ in question is expected to be reached for the first time at $E\tilde{T}_h$ and if $\tilde{y}$ matches the value of $Y^*(T)$ at this particular time (for $T = E\tilde{T}_h$), we obtain the first passage time T^* (intersect of the expected first-time realization of initial income and the threshold in Figure 3·1). Hence, as long as the first passage time has not been reached, the option value F_h is greater than the net value of current human capital V_h.[26]

[26] As in Dixit and Pindyck (1994, p. 160) the curves F_h, V_h have an upward slope. However,

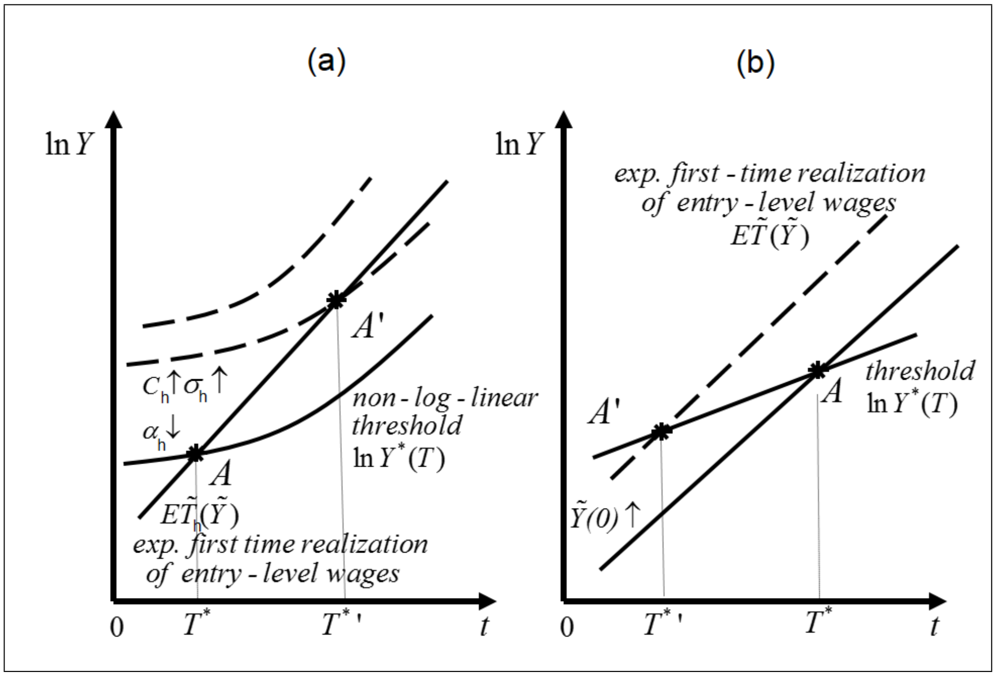

Figure 3·1: Earning Profile, Entry-Level Wage and Dynamics of Income

Proposition 6 *a) With the threshold $Y^*(T)$ from* (3.8)*, the expected first-time realization of initial income levels $E\tilde{T}_h$ from.* (3.10)*, and conditions*[27]

$$\frac{\beta_h}{\beta_h - 1}(r - \alpha_h)\bar{C}_h > \tilde{y}_0, \tag{3.11}$$

and

$$\bar{C}_h r > \bar{C}_h(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2) > C_h, \tag{3.12}$$

under certain conditions they can also decrease because in this model costs are accumulated.

[27] *Condition (3.12) seems restrictive. However, this sufficient condition reflects the simplifying assumption that schooling costs are constant for each year of schooling and the dynamics of the income process (3.3) do not change with more years of schooling. If income dynamics were positively related to the duration of schooling ($\alpha(T), \alpha' > 0$), this condition could be substituted by a more general simple condition. The explicit discussion of this condition and the implications are left for future research.*

there is an expected time of leaving school and entering working life $T^* = E(T_h) > 0$ *(first passage time). b) For each vector* $(\alpha_h, r, T^*, C_h, \tilde{y}_0, \tilde{\alpha}_h, \bar{C}_h)$ *that fulfils a) there is a marginal environment such that* T^* *is an implicit function of* $\alpha_h, C_h, \tilde{y}_0, \tilde{\alpha}_h, \bar{C}_h$ *and* r

$$T^* = T^*(\alpha_h, \tilde{\sigma}_h, C_h, \tilde{y}_0, \tilde{\alpha}_h, \bar{C}_h,\ r).$$

Proof. For a proof see Appendix B.3.2. ■

In Figure 3·1, a threshold that is higher than the expected entry-level wage reflects that learning costs during the education phase (before T^*) are not yet sufficiently compensated by the present entry-level wage, so the student prefers to stay in the educational system. In addition, condition (3.11) is important to understand the logic of the decision problem. The decision in favour of a given education level will only be positive if the minimum wage (no-education income $\tilde{y}_0$) is sufficiently small compared to education costs (3.11).[28] Further, the expected time of market entry (T^*) is just an indicator of what is expected to happen in future. Future development is partly random and therefore an unexpected exit from schooling can easily happen any time.

Determinants of Timing Decision In this section, we examine the most important and most frequently discussed determinants of the market entry decision. In particular, we look at the effects of risk, costs of schooling, and no-education income.

Proposition 7 *A larger risk* σ_h*, higher periodic education costs* C_h*, a flatter income profile and a lower no-education income level will lead to an increase in the expected*

[28] Both conditions are required for the existence of a solution to the problem. (3.11) is needed for the *threshold curve in* Figure 3·1 to start above the *entry-level* wage *curve.*

duration of schooling T^*

$$\frac{\partial T^*}{\partial \sigma_h} > 0, \quad \frac{\partial T^*}{\partial \alpha_h} < 0, \quad \frac{\partial T^*}{\partial \tilde{y}_0} < 0, \quad \frac{\partial T^*}{\partial C_h} > 0.$$

Proof. For a proof see Appendix B.4. ■

The influence of rising risk - measured by the volatility of revenues - on the time of market entry can be expected and is consistent with Hogan and Walker (2007), but deviates from the results of Groot and Oosterbeek (1992) and Hanchane et al. (2006). An increasing income risk will devalue the earning stream and hence will require higher compensation reflected by an increased threshold. As long as the additional net rewards of longer education can compensate for the rising threshold, students will stay in school.

Declining general income growth affects the benefits of education. Lower earnings growth will decrease the expected present value of schooling. Lower growth and hence less attractive earning track dynamics will only pay off if entry-level wages increase. With a sufficient marginal reward α_h, the required threshold can still be reached after more years of schooling. This new earning profile, characterized by a higher entry-level wage to compensate for less rapid income growth, justifies an even longer education. The effects of increasing α_h can also be described by another intuitively plausible story. If α_h increases, the ENPV of human capital increases as well (Figure 3·1a). As it is now easier to obtain the same value with lower investments, investments can be reduced.

The minimum wage level $\tilde{y}_0$ represents the no-education wage level when the schooling decision is made at $t = 0$. If the agent did not obtain any schooling he or she could start working for this entry-level wage $\tilde{y}_0$. In case of an increasing no education wage level, educational attainment will decrease (Figure 3·1b). This finding is intuitively expected. A rise in $\tilde{y}_0$ indicates (all else being equal) that no-education leads to higher income. The higher the no-education wage path, the less attractive a long education and the more attractive an early market entry will be.

As C_h denotes the flow of investment costs of schooling, the reaction $\frac{dT^*}{dC_h} > 0$ is not expected. In the standard approach, higher investment expenditure would increase the opportunity costs of education and would hence make education less profitable. Educational attainment would decline as a result. Therefore, this outcome can be regarded as a "tuition paradox". In this approach, the decision problem is different. As the costs of schooling increase, the student needs compensation from the market to stay in the system. As long as the market rewards the outcome of additional schooling sufficiently ($\tilde{\alpha}_h$ is sufficiently high), both curves would still intersect at a later time. In other words, the new earning profile promises a sufficiently higher earnings path to compensate for the increase in costs and justify even more education. According to empirical results provided by Heckman et al. (2008), increasing costs could be partially compensated by higher investment in schooling and a corresponding rise in entry-level wages. Higher costs may lead to longer education as long as the rewards are sufficient. The proposition is derived from comparative statics for marginal variations in a marginal environment around the solution.[29] If costs are too high and the market cannot sufficiently compensate, there is no intersect of the two curves in Figure 3·1. In this case, students would not decide to remain in school. Hence, the intuitively expected outcome of cutting short one's education when costs become too high can be also obtained as soon as the non-linear expansion of the threshold no longer allows for an intersection. This is one way to unravel the mystery of the tuition paradox.

[29] Hence, we remain in the inner solution of the model.

3.4 Sheepskin Effects and Large Shocks

"For two reasons, the dynamic nature of schooling suggests that the returns to education may include an option value. First, the return to one year of school may include the potential for larger returns associated with higher levels of formal qualification when the returns to school are not constant across all formal qualification levels. For example, finishing high school provides access to college, and attending college is a necessary first step for obtaining a college degree. [...]" [Heckman et al., 2006, p. 37].[30] Heckman et al. (2006) point to the multi-stage character of the process. The successful completion of one stage is necessary for proceeding to the next, and completing one formal education level results in an extra income premium. This so-called "sheepskin effect" shifts the income profile to a higher level. In particular, the sheepskin effect seems to be increasingly important in the recent empirical discussion, since recent findings support the hypothesis of nonlinearity in incomes which occur especially with high school and college completion.[31] Therefore, this section will model these additional elements of the education decision by extending the previous model and discussing discontinuities and nonlinearity in earning profiles.

Earning Profiles and Formal Qualification Levels For simplicity, we examine two levels of formal qualification.[32] For each level, both years of schooling and the achieved formal qualification determine the earnings profile. As the above model can

[30] The second reason to implement option theory in the human capital decision is already discussed in the above model: "[...] Second, when there is uncertainty about college costs or future earnings and when each additional year of schooling reveals new information about those costs or earnings, the full returns to schooling will include the expected value of newly revealed information that can be acted on." [Heckman et al., 2006, p. 37].

[31] See, e.g., Heckman (1995); Heckman et al. (2006, 2008); Denny and Harmon (2001); Skalli (2007); Ferrer and Riddell (2008); Silles (2008).

[32] Altonji (1993) examines education choice as a sequential choice that is made under uncertainy. Using a simple two-period model he estimates how variables that influence, e.g. tastes for school, payoffs to college, affect the expected return to a year of school.

be regarded as a model for only one level of formal qualification, we can consider more levels by simply symmetrically adding additional earning profiles. Specifically, for each level of formal qualification $i = 1, 2$, we assume specific costs $I_{h_i} \in \mathbb{R}$ depending on $C_{h_i} \in \mathbb{R}$ and a specific earning profile determined by entry-level wages $\tilde{Y}_i(t) \in \mathbb{R}$ and wage earnings $Y_i(t) \in \mathbb{R}$.

A student observes that for each formal qualification level i a year of additional schooling will increase his or her *entry-level* wage $\tilde{Y}_i$ according to the geometric Brownian motion[33]

$$d\tilde{Y}_i = \tilde{\alpha}_{h_i}\tilde{Y}_i + \tilde{\sigma}_{h_i}\tilde{Y}_i d\tilde{W}_{h_i}, \quad \text{for} \quad t < T_i,$$

where T_i denotes the years of schooling required to attain the formal qualification level i (e.g. a secondary education program may last for four years, hence $T_2 = 4$). Note that each level of formal qualification has its own drift $\tilde{\alpha}_{h_i}$ and volatility $\tilde{\sigma}_{h_i}$, which are positive and constant.

The *dynamic development of income* Y_i during working life for each level of formal qualification i is highly uncertain. Uncertain fundamental events like the threat of unemployment or disease, or opportunities like sudden promotions or job offers that fundamentally affect a career, are important and specific elements of an income profile. These events often trigger large downward or upward leaps in income. In order to account for these large random impacts, we distinguish marginal risk and large random events that result in a stochastic shock. On the one hand, there are marginal fluctuations in income growth, usually known as the marginal risks. In real life these may be random changes in wage growth such as 3% in one year and 2% in the other. On the other, new jobs and opportunities may lead to strong upward shifts

[33]Each level of formal qualification is modeled symmetrically based on the referencing model above. Therefore, see also (3.2) for only one level of formal qualification.

in income, or sudden dramatic and non-marginal threats such as unemployment or an illness can trigger large downward shifts in income. These opportunities and threats are often related to the formal education achievement. Typically, university graduates do not just learn more and become more productive due to the content of their education. They are more likely to meet influential people, to enter powerful networks, and to develop the personality that is necessary to match the sophisticated requirements for a leading position in a company. Further, low-skilled labour does not have to be less productive. Workers with lower skills can be more easily substituted, they are more likely to lose their job during an economic downturn, and may even be more at risk of illness due to working conditions. However, such opportunities or threats have an economic value and affect the value of a formal qualification, even if the realization of such income jumps remain random. Positive and negative jumps as elements of a random income dynamics are best described as stochastic shocks. So far, there is no literature that analyses the effects of such large stochastic shocks and fundamental threats and opportunities related to the education decision.

The appearance of these large opportunities and threats can be described by another stochastic process. For the present case, we describe the development of income for each formal education level $i = 1, 2$ as an Ito-Lévy Jump Diffusion process[34]

$$\frac{dY_i}{Y_i} = \alpha_{h_i} dt + \sigma_{h_i} dW_{h_i} + \int\limits_{U_{h_i}} z_{h_i} N_{h_i}(t, dz_{h_i}) \quad \text{for} \quad T_i < t.$$

While in the real world an earnings profile would not be linear and decrease at the end of a working life, we keep matters simple and assume $\alpha_{h_i} \in \mathbb{R}_+$ to be

[34] A more general formulation of this process can be found in Oksendal and Sulem (2007). They describe under which conditions a solution to these SDE exists and discuss some characteristics. For our purpose we assume that the existence conditions are fulfilled. A further discussion of Lévy processes and their characteristics can be found in, e.g. Applebaum (2009) and Cont and Tankov (2004).

constant. The first part of the stochastic process is a geometric Brownian motion with a constant marginal risk represented by volatility σ_{h_i}, followed by a jump part $\int\limits_{U_{h_i}} z_{h_i} N_{h_i}(t, dz_{h_i})$. $N_{h_i}(t, dz_{h_i})$ denoting the Poisson process with intensity λ_{h_i}. Hence, non-marginal jumps which occur at a random time with an uncertain step height out of U_{h_i} are accumulated. $U_{h_i} \subseteq (-1, 0) \cap (0, \infty)$ itself is a Borel set whose closure does not contain 0.[35]

Once working life begins earning profiles are determined within the limits of the random process related to each achieved formal qualification. The Expected Net Present Values of human capital V_{h_i} when leaving school in T are given by[36]

$$V_{h_1} = \frac{Y_1(T)}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1 + z_{h_1}) - z_{h_1}\right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1}\right)} - \int\limits_0^T C_{h_1} e^{r(T-t)} dt - \bar{C}_{h_1}$$

and

$$V_{h_2} = \frac{Y_2(T)}{\left(r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) - \int\limits_{U_{h_2}} \left[\ln(1 + z_{h_2}) - z_{h_2}\right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2}\right)} - \int\limits_{T_1}^T C_{h_2} e^{r(T-t)} dt - \bar{C}_{h_2}.$$

f_h denotes the function $f_h(z_h) = ln(1 + z_h)$ and υ_{h_i} refers to the Lévy measure of the Poisson process N_{h_i}.

[35] We have to assume U_{h_i} having a lower boundary not lower than -1, because, as will be seen later, the solution to the differential equation dY_i has a solution only for $z_{h_i} > -1$.

[36] See Appendix B.5.2.

Option Value of Completing a Formal Qualification Entering a higher education program often requires the prior successful completion of a lower educational level. For instance, if a student wants to start a college degree, he or she needs to graduate from high school beforehand. Furthermore, finishing each formal education level provides an income premium, the sheepskin effect. We can even keep thinking about this effect when we account for the academic reputation of different schools. Hence, looking at a sequential decision problem, not only the option value of completing another year of schooling but also the option of a sheepskin effect should be included in the decision.

"Our findings suggest that part of the economic return to finishing high school or attending college includes the potential for completing college and securing the high rewards associated with a college degree. Both sequential resolution of uncertainty and non-linearity in returns to schooling can contribute to sizeable option values." [Heckman et al., 2006, p. 7].

In our model we have only two formal education levels. Hence, as the second level is the highest to be considered, the option value, F_{h_2}, includes the sheepskin effect S_2 of successfully graduating from this level. This sheepskin effect pushes up the entry-level wage as a reward for graduation and consists of all income effects provided by the new earning profile connected to graduation. For simplicity, we assume that the sheepskin effect is discounted linearly, so that the time effect, which is the derivative of F_{h_2} with respect to t, has a value of $\frac{\partial F_{h_2}}{\partial t} = rS_2$. Applying Ito's Lemma and the Bellman equation results in

$$rS_2 + \tilde{\alpha}_{h_2} \tilde{Y}_2 \frac{\partial F_{h_2}}{\partial \tilde{Y}_2} + \frac{1}{2} \tilde{\sigma}^2_{h_2} \tilde{Y}^2_2 \frac{\partial^2 F_{h_2}}{\partial \tilde{Y}_2 \partial \tilde{Y}_2} - rF_{h_2} = 0.$$

This is a second-order inhomogeneous differential equation with a free boundary and has the solution[37]

$$F_{h_2} = K_{h_2} \tilde{Y}_2^{\beta_{h_2}} + S_2,$$

with $K_{h_2} \in \mathbb{R}$ constant. Hence, for the exponent we obtain $\beta_{h_2} = \frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}^2_{h_2}} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}^2_{h_2}}\right)^2 + \frac{2r}{\tilde{\sigma}^2_{h_2}}}$, which is similar to (3.9).

Knowing the option value of secondary education, we are able to derive the option value of primary education F_{h_1}. For each $t \in [T_1, T_2]$ F_{h_1} includes the discounted option value F_{h_2} and the discounted sheepskin effect S_1. Hence, the time effect of these two components is $\frac{\partial F_{h_1}}{\partial t} = r(S_1 + F_{h_2})$. Applying Ito's Lemma and the Bellman equation results in

$$r\left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}}\right) + \tilde{\alpha}_{h_1} \tilde{Y}_1 \frac{\partial F_{h_1}}{\partial \tilde{Y}_1} + \frac{1}{2} \tilde{\sigma}^2_{h_1} \tilde{Y}_1^2 \frac{\partial^2 F_{h_1}}{\partial \tilde{Y}_1 \partial \tilde{Y}_1} - r F_{h_1} = 0.$$

This is the so-called inhomogeneous Euler differential equation which has the solution[38]

$$F_{h_1} = K_{h_1} Y^{\beta_{h_1}} + r\left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}}\right), \tag{3.13}$$

where the respective positive and negative roots obtained for the homogenous differential equation are $\beta^1_{h_1} = \frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}^2_{h_1}} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}^2_{h_1}}\right)^2 + \frac{2r}{\tilde{\sigma}^2_{h_1}}} > 1$ and $\beta^2_{h_1} = \frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}^2_{h_1}} - \sqrt{\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}^2_{h_1}}\right)^2 + \frac{2r}{\tilde{\sigma}^2_{h_1}}} < 0$. Note that (3.13) entails components of F_{h_2}, namely K_{h_2} and β_{h_2}, the threshold of the second formal education level and both sheepskin effects S_1 and S_2.

[37] See Appendix B.5.2.

[38] In order to obtain the solution transform the inhomogenous differential equation in an inhomogenous linear differential equation with constant coefficients. This can be done with $\tilde{Y} = \exp(t)$. Next, find a solution to this equation by means of the characteristic polynomial and variation of constants. Finally, transform the solution back to the original variable.

Entry Thresholds of Formal Qualifications In order to derive the entry threshold for each education level, we proceed according to the simple case of one education level. Hence, we use the net values of human capital and by comparing them with the option values we determine the threshold curves $Y_i^*(T)$. Finally, the boundary conditions lead us to the threshold for the second education level Y_2^*.

Proposition 8 *For education costs I_{h_2}, a sequence of increasing earning levels through schooling, described by the geometric Brownian motion $\tilde{Y}_2$, and an earning dynamics after market entry Y_2, described by a Ito-Lévy Jump Diffusion, the threshold $Y_2^*(T)$ that triggers the start of the earning process is*

$$
\begin{aligned}
Y_2^*(T) &= \begin{pmatrix} r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) \\ - \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2}\right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2} \end{pmatrix} \frac{\beta_{h_2}}{\beta_{h_2}-1} \left(I_{h_2} + S_2\right), \\
\textit{with} \quad r &> \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) + \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2}\right] \upsilon_{h_2}(dz_{h_2}) + \alpha_{h_2}.
\end{aligned}
$$

Proof. For a proof see Appendix B.5.3. ■

For the first education level $i = 1$ we obtain a non-linear problem, so that the solution to the threshold can only be obtained as an implicit function.

Proposition 9 *For education costs I_{h_1}, a sequence of increasing earning levels through schooling, described by the geometric Brownian motion $\tilde{Y}_1$, and an earning dynamics after market entry Y_1, that follows an Ito-Lévy Jump Diffusion, the set of points $Z = (r, z_{h_1}, \alpha_{h_1}, \tilde{\alpha}_{h_1}, \tilde{\alpha}_{h_2}, \tilde{\sigma}_{h_1}, \tilde{\sigma}_{h_2}, S_1, S_2, C_{h_1}, \bar{C}_{h_1}, \lambda_{h_1})$ forms a submanifold of the $\mathbb{R}^{13}$ with dimension 12.*

Proof. See Milnor (1997), p. 11. ■

Using Proposition 9**,** we can show that there is an implicit function for the threshold Y_1^* which depends on Z, i.e., all parameters determining the net present and option value.

Proposition 10 *For each vector*

$$Z_0 = (r_0, z_{h_{10}}, \alpha_{h_{10}}, \tilde{\alpha}_{h_{10}}, \tilde{\alpha}_{h_{20}}, \tilde{\sigma}_{h_{10}}, \tilde{\sigma}_{h_{20}}, S_{10}, S_{20}, C_{h_{10}}, \bar{C}_{h_{10}}, \lambda_{h_{10}}),$$

that fulfils

$$H \ : \ = \frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{Y_1^*}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1 + z_{h_1}) - z_{h_1} \right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)} \\ -r \left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}} \right) - I_{h_1} = 0,$$

there is a marginal environment around this vector such that Y_1^* is an implicit function of Z

$$Y_1^* = Y_1^*(Z). \tag{3.14}$$

Proof. For a proof see Appendix B.5.2. ■

As we can see from (3.14), the threshold depends on both sheepskin effects, the frequency and direction of opportunities and threats and several other parameters that stem from the two option values F_{h_1} and F_{h_2}.

Impact of Sheepskin Effects and Long-Term Incentives In the previous section, we determined the two thresholds that trigger the end of schooling for each level of formal qualification. In particular, we are able to show that risk, irreversibility and discontinuous elements like sheepskin effects are an important ingredient of the decision problem. In this section, we examine the impact of changes in sheepskin effects at both levels of education. In particular, we show how sheepskin effects provide a kind of long distance incentive for the today's education decision. Even if this added bonus is in the future, its expected realization is part of the option value and hence affects today's education decision. We can find these effects even during the first formal education level by looking at the respective threshold.

Proposition 11 *An increase in the sheepskin effects S_1 and S_2 of both formal qualification levels increases the threshold to leave the first formal qualification level*

$$\frac{\partial Y_1^*}{\partial S_1} > 0 \text{ and } \frac{\partial Y_1^*}{\partial S_2} > 0.$$

Proof. For a proof see Appendix B.5.4. ■

A larger sheepskin effect means that finishing the respective formal education level has a greater value. Leaving school, however, requires a higher entry threshold due to increased opportunity costs of leaving school. Hence, the decision to stay in the first education period (high school, for instance) is not only determined by the value of obtaining the first formal qualification but also includes the opportunity to start and finish the second formal education level. The student knows that once he or she finishes the second formal level (college, for instance) he or she can expect to earn an additional, possibly even greater sheepskin effect. Both components represented by the sheepskin effects S_1 and S_2 increase the option value of additional education and hence the value of staying in school. Even if the length of time until the decision has to be taken is long, the effect remains a component of the decision. The sheepskin effect of a college degree is an incentive to complete high school.

Impact of Marginal Risk and Stochastic Shocks Marginal risk covers marginal fluctuations in income growth, usually depicted by $\tilde{\sigma}_i$. Stochastic shocks are strong (more than marginal) upward or downward shifts in income. An upward shift may be an unexpected opportunity that affects the related income profile. A college graduate typically is more likely to get a high-paid job than a high school graduate. Someone with lower formal qualification may be unemployed more frequently and hence faces a more severe threat of downward income jumps. These positive and negative jumps as elements of income dynamics are best described as stochastic shocks. Here we analyse the effects of an increasing frequency of these jumps on the education decision. We start with the effects of marginal risk and without loss of additional insights, only look at the second level of formal qualification.

Proposition 12 *(i) An increase in marginal risk $\tilde{\sigma}_{h_2}$ of the second formal qualification level increases the threshold Y_2^*. Hence,*

$$\frac{\partial Y_2^*}{\partial \tilde{\sigma}_{h_2}} > 0.$$

(ii) An increase in the frequency of jumps λ_{h_2} reduces the threshold to leave the second formal qualification level, under the condition that positive jumps outweigh the negative ones, so that $\int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} g_{h_2}(dz_{h_2}) + \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2}\right] g_{h_2}(dz_{h_2}) > 0$ with g_{h_2} being the probability distribution of jump heights. Hence,

$$\frac{\partial Y_2^*}{\partial \lambda_{h_2}} < 0.$$

Proof. For a proof see Appendix B.5.4. ■

Looking at (i), the effect of marginal risk, we can see that an increasing volatility of the motion of the entry-level wage leads to a larger required threshold to leave school. For a simpler model this result was already derived in Proposition 7 and in Hogan and Walker (2007). In (ii), for the condition 'positive jumps outweigh the negative ones', the random process offers more opportunities than threats.[39] In this case, a higher frequency of jumps implies that promotions, job offers and other events leading to non-marginal upward income shifts, become more likely once a person is in the market. Therefore, with a higher frequency of chances offered by the market, staying at school becomes less attractive. Improved market opportunities cannot be realized as long as a student stays at school. Mark Zuckerberg and Steve Jobs are well-known examples. They saw huge opportunities that they could only seize once they had left school and entered the market. These enormous opportunities acted as an incentive for them to start working. Leaving school was expected to be more profitable than continuing education. If such opportunities are a characteristic of a formal qualification and hence elements of a sheepskin-reward, they become part of the option value and make completion of formal qualification more valuable.

3.5 Conclusion

Education is a multi-stage investment and the realization of the various stages takes time. We consider two phases, a pure investment phase followed by a second earning phase. While in the investment phase each period's investment improves the outcome of the project, it is uncertain how long a student will have to invest and how much he or she will have to accumulate in order to maximize the education value. Hence, the question we answer is, how long (first passage time) and how much we can expect to invest during this kind of multi-stage sequential education process.

Recent literature shows that real option theory can be applied to these ques-

[39] This condition may be related to one's individual education or personality, or it may be related to general changes in business conditions such as an economic upswing or downturn.

tions in order to take into account uncertain time processes and irreversibility in human capital accumulation decisions. Based on the modelling suggested by Hogan and Walker (2007), we extend their framework by (1) adding accumulated education costs and determining the expected duration of schooling, (2) considering complete earnings profiles including entry-level wage, sheepskin effects and earning dynamics, and (3) discussing the option value of schooling introducing potential career opportunities or threats of unemployment modeled as major uncertain events connected with a particular education achievement. Marginal risk covers marginal fluctuations in income growth, usually depicted by the variance. Stochastic shocks are strong (more than marginal) upward or downward shifts in income. As these kind of large sudden events are an important ingredient of income profiles connected to qualification levels, we account for this phenomenon by extending the standard version of the model by an Ito-Lévy Jump Diffusion process.

From comparative statics, we obtain: (i) With an increase in education costs the student may stay in the system as long as the increasing costs of schooling are compensated sufficiently by the market. (ii) A sheepskin effect may produce an extra income premium, and completion of a formal education is often a necessary precondition for moving to the next level. Both facts describe a discontinuous jump in rewards once a student achieves a formal qualification. This leap increases the option value of additional education and hence the value of staying in school. Sheepskin effects, even for future levels of qualification and even if the time to completion is long, remain an encouraging component in the decision to remain in school longer. (iii) With respect to the effects of stochastic major events that we analyse, staying in school becomes less attractive if the frequency of such uncertain events offered by the market increases and if there are more overall opportunities than threats. In this case, the uncertainty in the market is positive and students will want to seize these positive opportunities. If fewer threats and increasing opportunities go along with an increasing formal level of qualification, the option value of achieving higher levels of education increases.

Chapter 4

Uncertainty and Conflict Decision

This chapter is a joint work with Thomas Gries which was published as a working paper No. 2014-05 in the CIE Working Paper Series. It is a slightly revised version of the paper that was published in the Conference Paper Series "Beiträge zur Jahrestagung des Vereins für Socialpolitik 2012: Neue Wege und Herausforderungen für den Arbeitsmarkt des 21. Jahrhunderts", Session: Conflict and Disputes, No. C20-V3.

4.1 Introduction

The last contribution of this doctoral thesis extends the previous by allowing for a large stochastic variability during the waiting period. In the context of social conflicts, a rebel decides about whether and when to attack the oppressive government in order to change the status quo. This decision is particularly determined by large stochastic events generated by, e.g. the government. Hence, the focus of this contribution lies on the effect of non-marginal stochastic events during the latent conflict phase, where the outbreak of conflict has not taken place yet. Similar to the previous chapter, a real option model is used to determine the required benefit that the conflict needs to generate, and the optimal time of attack. The results show that stochastic shocks have an ambiguous effect, depending on whether marginal or non-marginal. That is, marginal stochastic variability may extend the non-violent period while non-marginal stochastic variability leads to an earlier attack.

Almost no decision problem bears more dramatic and large uncertainties than the decision to launch a violent conflict. While empirical studies on the causes and consequences of social conflicts are manifold, consistent closed formal theories addressing uncertainties in the decision to launch a conflict remain rather limited,[40] on both the macro and the micro level.[41] Hence, in this paper we ask how uncertainty affects the decision to launch a conflict. Using option theory, we can distinguish two types of uncertainty. For a rebel, an increase in marginal uncertainty which can be connected to typical fluctuations in the economy may give hope and lead him or her to postpone the violent action. In contrast, an increase in massive threats and major uncertain events, such as extensive oppressive government actions, will have the opposite effect and encourage an earlier outbreak of conflict. The implications are straightforward: conditions, whether exogenous or deliberately provoked that generate large uncertainties affect conflict decisions and the duration of a non-violent period that could be used to find a long-term solution to the conflict.

In the literature, it is well recognized that turning to violence can be a rational choice for, e.g. an oppressed group.[42] In particular, discrimination, repressive gov-

[40] See the survey by Sandler and Enders (2004) and Blattman and Miguel (2010).

[41] For the case of conflicts with strategically interacting groups such as well-organized rebel groups fighting to overthrow the government, game theory approaches show that conflicts can have economic roots and may be the result of strategic interactions between conflict partners. In particular, conflicts between two competing agents occur due to a lack of property rights (Skaperdas, 1992), incomplete and insufficient information about the other party's relative military power (Fearon, 1995; Yared, 2010), or due to the agent's inability to estimate the opponent's ability to win (Powell, 2002, 2006). These scenarios seem suitable for explaining certain kinds of social conflict, yet they shed light only on explicit bilateral interaction with mutual strategic behavior of clearly defined and strictly controlled conflict parties, such as a well-organized, homogeneous group of rebels engaged in a game with the government. In this case, both parties know the possible set of actions before the opponent turns to violence, and take them into account.

[42] Empirical investigations assess that attacking in the context of terrorism is a rational choice. See, e.g., Anderton and Carter (2005), Finkel and Muller (1998), and Weede and Muller (1998).

ernment actions, waves of political persecution, arbitrary detention, torture of group members, and bad living conditions may cause frustration and lead to violent action of all kinds. Launching a violent conflict may become an instrument for improving conditions for the rebel or his or her social group (Tornell, 1998; Gould and Klor, 2010), or to maintain or acquire power (Besley and Persson, 2011). As Blomberg et al. (2004) and Caplan (2006) claim, such conflicts may even be a rational outcome if there is no other way to bring about drastic institutional change. Even if the decision to attack is based on a comparison of the perceived benefits and costs of rebellion (Collier and Höffler, 1998), and even if the goal of maximizing income or welfare prevails (Grossman, 1991), it is highly uncertain whether keeping the peace for a while longer may also improve conditions. Similarly, it is highly uncertain whether turning to violence will improve conditions for the attacking group in the aftermath. Uncertainty can trigger violence, just as it can encourage the group to wait for an ad-hoc improvement in conditions.

First thoughts about uncertainty in theoretical modelling in this context come from Morrow (1985), who claims that war decisions are based on the actors' utility of uncertain outcomes. This approach is extended by Collier and Höffler (1998) and Besley and Persson (2011). The role of uncertainty in formal conflict theory is rather rudimentary, even if nothing is more likely to trigger a conflict than an unforeseeable major event, and nothing is more uncertain than the outcome of a conflict once it is triggered.[43] Empirical approaches, however, have started to assess how large - not only government-induced - uncertainties, such as disasters, affect e.g. terrorist activities. Major shocks cause additional uncertainty and may strongly affect or even help to overthrow existing conditions. They impact the motivation

[43] For instance, in the aftermath of a conflict rebels may become heroes, or they are just as likely to be killed or their families exposed to even more severe repression.

to become violent and can lead to an outbreak or the escalation of social conflict. For instance, Brancati (2007) shows that earthquakes may be a driving force for intrastate conflicts, in that they increase competition for scarce basic resources. He also concludes that the effect is greater for higher-magnitude earthquakes that strike more densely populated areas of countries with lower gross domestic products and pre-existing conflicts. Berrebi and Ostwald (2011) extend this approach by taking into account both earthquakes and hurricanes and show that terrorism may be a consequence of natural disasters. Besides the effect of natural disasters, economic disasters such as the financial crisis may encourage terrorist activities due to their destabilizing effects (Gries and Meierrieks, 2013).

Hence, in this theoretical contribution we analyse how variability created, e.g. by governments, natural disasters, or economic crises affect the assessment of conflict benefits and hence the outbreak of a conflict. In line with Blomberg et al. (2004), we illustrate our discussion using the example of uncertain government repressions that may lead to frustration and finally to the outbreak of conflict. In particular, the government may increase the intensity and severity of discriminating and repressing activities against a particular social group. Here, variability created by the government is taken as given by the rebel, and we analyse its effect on the rebel's reaction. Is higher variability a driving force for conflict escalation, or does it encourage rebels to remain peaceful? Does, e.g. a higher frequency of oppressive policies or more severe government action deter the rebel group, or does it encourage it to take action and change conditions through violent means?

To analyse the effect of variability on the decision to turn to violence we choose a real option approach because it considers the value of (peacefully) waiting until (violent) action is taken. Since uncertainty is evaluated in the course of time we are able to find an optimal timing decision. We can distinguish two types of variability, namely marginal stochastic shocks that indicate the variability of general living conditions (which are sometimes better, sometimes worse), and non-marginal stochastic shocks for sudden major threats (more repressive actions).[44] More importantly, we show that these two types of uncertainty lead to opposite effects on the outbreak of conflict.

Why is this important to know? Because we can now evaluate how government actions, such as increasing threats, will stabilize or destabilize a latent conflict. While conventional option modelling[45] suggests that higher marginal stochastic variability, with the interpretation of more economic variations, encourages the rebel to remain peaceful for longer, stochastic large threats created by the government bring the attack forward. Hence, the decision of whether and when to attack strongly depends on the frequency and size of uncertain events. More threats will abbreviate the expected peace period and reduce the chance of a peaceful long-term solution to the conflict.

4.2 Model

4.2.1 Model Idea

Many attacks are individual or small-group violent actions by agents. Irrespective of group dynamics or psychological, ethnic, or sociological reasons, we focus on the idea that starting a conflict can be regarded as an investment in a better but uncertain future.[46] An attack is not something that unexpectedly enters the mind of a decision-

[44] More precisely, as we use an Ito-Levy jump diffusion process that extends the geometric Brownian motion by a jump component, we can analyse (i) marginal stochastic shocks that refers to the variance in standard modelling of the Brownian motion, and (ii) non-marginal stochastic shocks caused by a frequency of jumps of unknown size.

[45] Here "conventional" means modelling using a Brownian motion.

[46] Besides the economic analysis of social conflicts there is an extensive discussion on psychological and sociological reasons for, e.g. terrorist attacks. See, e.g., Muller and Opp (1986), and Victoroff

making individual. Rather, it is the result of a dynamic process in which the current path of development of the economic and social situation (status quo) is evaluated and compared to the expected path of development after a potential attack, including conflict costs and all potential threats and opportunities (Grossman, 1991). If current conditions are dissatisfying, a latent conflict exists and violence may be considered. One of the potential outcomes of such a situation is an attack. For instance, a rebel who plans a terrorist attack will only carry out the plan if he or she expects to have an overall positive effect. If the assassination is successful, repression may stop and reforms could lead to more political participation or economic improvements. As rebels never know when and how strong repression will be, they take the uncertain environment created by the government as given and decide whether, and if so, when it is optimal to attack. Even if an immediate attack may have some benefits, it is possible that a non-violent strategy comprising a potential later attack is the better option. Rebels act rationally (Caplan, 2006); they decide whether to invest immediately and pay the price by launching the conflict (attack), or to maintain the status quo, at least for a while. Since conditions are highly uncertain and may change even without an attack, sometimes a simple waiting period may be more beneficial. Hence, the hope of non-violent change or the expectation that a later attack (by someone else) is more beneficial may postpone the attack. As time goes on, rebels repeatedly consider their living conditions, which in bad times become worse, and they repeatedly decide whether to arrest this process of deterioration by violent means. As each moment's conditions determine this decision, it is a sequential decision in time. The decision is also irreversible – once the attack is carried out, there is no turning back. All consequences have to be accepted, and the freedom and flexibility to choose more moderate strategies to solve the conflict are no longer

(2005).

present. Hence, with sequential decisions, high uncertainty, and irreversibility as major components of the decision problem, real option theory is an appropriate methodology. In order to capture large uncertain shocks, we extend the conventional (Brownian motion) model by using an Ito-Lévy Jump Diffusion process.[47] Unlike the Brownian motion, which is the 'workhorse process' in real option theory, this more general process allows for evaluating parameters characterizing the frequency and the scale of major events as well as the regular risk measure which is the volatility of the geometric Brownian motion. With an Ito-Lévy Jump Diffusion process, we are therefore able to analyse substantially different risk components of a stochastic process and we can show that they have opposite effects on the decision to turn violent. In this model, the rebel maximizes his or her present discounted net value of the benefit of an attack (including the value of flexibility) by deriving a conflict threshold that determines the benefit level required to trigger the violent outbreak. Randomly reaching this threshold represents the straw that breaks the camel's back. The decision is determined by a sequential comparison of the net present value of the benefits of a potential attack with the value of postponing an attack to a later date. Having established this triggering threshold, we can also determine the first passage time, that is, the expected time of attack. Even if our sequential process identifies an expected time of conflict outbreak, the model is also able to suggest that a sudden random change in conditions may also lead to an unexpected attack at any moment.

4.2.2 Benefits of Conflict

As the attack is expected to change the social, economic, and political conditions created by the oppressing government, there are potentially two periods in conflict

[47] The importance of jump diffusions was first recognized in financial economics. For instance, Merton (1975) derived a price for an European option similar to the Black Scholes formula. In the course of time some extensions of the Merton approach followed.

evaluation with two sets of conditions: first, the current period with a set of current conditions associated with a path of non-violent but dissatisfying development; and second, a new set of conditions in the period after the attack that are expected to generate a better path of development. Both elements determine the evaluation of total benefits of the conflict and eventually, the decision to launch it in order to generate a structural break in living conditions of the rebel or his or her group.

Current Conditions and Path of Development In this model, current conditions lead to a time path that is not satisfying for a certain social group. A harsh set of conditions for this group provokes resistance and a start of a latent conflict between an oppressive government and rebels. Increasing repression, worsening economic restrictions or discrimination, growing inequality of opportunities, and an increasing threat of persecution may lead to greater frustration in the face of deteriorating opportunities, and will eventually increase the propensity to turn violent. Further, with each additional moment of waiting and not attacking, the worsening welfare of the rebels may generate an increasing current benefit of conflict by rate $\tilde{\alpha}_s \in \mathbb{R}_+$. In other words, as the attack is a potential action, deteriorating living conditions increase the benefits of an attack. The expectation of a deteriorating current time path of welfare produces a sufficiently bleak outlook as to make an attack increasingly beneficial. However, there are marginal fluctuations due to small variations in the economic and political situation that are described by the volatility $\tilde{\sigma}_s \in \mathbb{R}_+$. In addition, government's repressive actions are highly uncertain to the rebel and may lead to a dramatic and non-marginal change in conditions from one moment to the next. These non-marginal stochastic shocks have an intensity $\tilde{\lambda}_s > 0$ and increase the benefits of conflict by a random amount $u_s \in \tilde{U}_s \subseteq (0, \infty)$. In particular, major disastrous events have negative effects on rebel's welfare and prospects, so that the current benefit of conflict may suddenly increase significantly. In order to consider these fundamental threats or great opportunities, we describe the development of current benefits of a potential attack during the period before the conflict is launched as an Ito-Lévy Jump Diffusion process.[48]

Definition 13 *Let* $\tilde{U}_s \subseteq (0, \infty)$ *be a Borel set whose closure does not contain* 0. *Furthermore, let* $\tilde{W}_s$ *be a standard Wiener process,* $\tilde{N}_s$ *a Poisson process with intensity* $\tilde{\lambda}_s$ *and constants* $\tilde{\alpha}_s, \tilde{\sigma}_s \in \mathbb{R}_+$. *Then the Ito-Lévy Jump Diffusion process* $\tilde{B}$, *indicating the benefits of a potential attack at time* t *during the period before the conflict at* T, *is defined by the following SDE for* $\tilde{B}(t) \in \mathbb{R}$

$$\begin{aligned} d\tilde{B} &= \tilde{\alpha}_s \tilde{B} dt + \tilde{\sigma}_s \tilde{B} d\tilde{W}_s + \tilde{B} \int_{\tilde{U}_s} u_s \tilde{N}_s(t, du_s) \qquad (4.1) \\ \text{for} \quad 0 &< t < T \quad \text{and} \quad \tilde{B}(0) = \tilde{b}_0. \end{aligned}$$

This stochastic process includes a continuous and a discontinuous part through a combination of a geometric Brownian motion and a compound Poisson process, which models exceptional stochastic events through the integral $\int_{\tilde{U}_s} u_s \tilde{N}_s(t, du_s) > 0$. The integrand u_s denotes the step height of jumps which is uncertain but limited by $\tilde{U}_s$.[49] Note that during calm periods where the government does not undertake any large repressive actions the rebel's welfare slowly becomes worse due to structural conditions which will lead to a continuous increase in the benefits of an attack. Should the government threaten the rebel or his or her group significantly, the benefits of conflict will increase by a random and non-marginal amount.

[48]The Ito-Lévy Jump Diffusion process is a special case of geometric Lévy processes. For further information about Lévy processes see, e.g., Oksendal and Sulem (2007) or Applebaum (2009).

[49]For a graphical illustration of Jump Diffusions, see, e.g., Cont and Tankov (2004), pp. 71.

Conflict Benefits in the Aftermath Since conflict is assumed to pay off somehow, an attack generates a structural break with a new set of better economic, social, or political conditions afterwards. The rebel's increasingly dissatisfying current situation increases the benefits of conflict so that once the attack has been carried out, the rebel's living conditions are expected to improve by rate $\alpha_s \in \mathbb{R}_+$, so that the resulting benefits of conflict can be realized in the aftermath. Returning to our example, successfully assassinating an oppressive leader may lead to political and economic reforms that, in the next step, improve the welfare of the rebel. Hence, even if high uncertainty is involved, carrying out an attack is expected to lead to a satisfactory improvement in the rebel's social environment.

The path of future benefits of conflict is highly uncertain. A new process of uncertain developments begins, and although living conditions may be expected to improve on average, changes due to usual economic fluctuations that are captured by a volatility $\sigma_s \in \mathbb{R}_+$ and unforeseen repressing events caused by the government with intensity $\lambda_s > 0$ and size $z_s \in U_s \subseteq (-1, 0)$ may take place and must be considered when evaluating the benefits of an attack. For instance, the government may become even more oppressive, the rebel may be caught or even tortured, and reforms, counterattacks, or military coups may fail. Since developments of future benefits in the aftermath of an attack incorporate fundamental threats, we model them as another Ito-Lévy Jump Diffusion process.

Definition 14 *Let $U_s \subseteq (-1, 0)$ be a Borel set whose closure does not contain 0. Furthermore, let W_s be a standard Wiener process, N_s a Poisson process with intensity λ_s and constants $\alpha_s, \sigma_s \in \mathbb{R}_+$. Then the Ito-Lévy Jump Diffusion process $B(t) \in \mathbb{R}$, indicating the future benefits in the aftermath of an attack, is defined by means of the following SDE*

$$dB = \alpha_s B dt + \sigma_s B dW_s + B \int_{U_s} z_s N_s(t, dz_s) \quad (4.2)$$
$$\text{for} \quad T < t \quad \text{and} \quad B(0) = b_0.$$

We have to assume a lower boundary to be -1 because, as will be seen later, the solution for dB only exists for $z_s > -1$.

While the first part of the stochastic process is again an increasing geometric Brownian motion, the second part, $\int_{U_s} z_s N_s(t, dz_s) < 0$, allows for fundamental repressing and unpredictable events in the aftermath of the conflict.

4.2.3 Value of Conflict and Option Value of Peacekeeping

Net Present Value of Conflict As conflicts may lead to an improvement in living conditions, an attack enables the realization of potential benefits of conflict. Once the attack is carried out, the dynamic development of benefits is given within the limits of a random process. The economic value of conflict consists solely of its future benefit stream. Each dynamic development of benefits generates its own value of conflict. For an investor the gross and net value of conflict V^{gross} is determined by the expected present value of the benefit stream in the aftermath.[50]

Proposition 15 *Let B be the benefit stream after the conflict described by Ito-Lévy Jump Diffusion process in (4.2), υ_{s_2} the Lévy measure and r the risk-free interest*

[50] A detailed solution to the SDE and the derivation of the expected value of the Ito-Lévy Jump Diffusion process, which is used for determining the gross value of conflict is presented in Appendix C.1.

rate. Furthermore, assume

$$r > a_s + \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) + \int\limits_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s),$$

where $f_s(z_s) = ln(1+z_s)$*. Then the gross value of benefits is*

$$V_s^{gross}(T) = \frac{B(T)}{\left(r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_{s_2}(dz_s) - \int\limits_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s) - a_s\right)}.$$

Hence, with conflict costs $I_s \in \mathbb{R}$ *the ENPV of conflict is*

$$V_s(T) = V_s^{gross}(T) - I_s.$$

Proof. For a proof see Appendix C.2. ■

Note that, for simplicity, the rebel has an infinite lifespan. In our example, an attack would enable long term political participation of a particular social group. Furthermore, we can see that the net present value depends on the sum of repression undertaken by the government after the conflict,

$$\int\limits_{f_s^{-1}(U_s)} z_s \upsilon_{s_2}(dz_s) - \int\limits_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s) < 0.$$

This additional component acts as an element of the discount parameter that occurs only after accounting for large and non-marginal risk. If we had used the geometric Brownian motion to describe the benefits of conflict, we would not obtain this parameter. This result is particularly convincing for our problem. The more repressive a government remains in the aftermath of the conflict, the lower the net present value of conflict and the less beneficial an attack.

Option Value of Peacekeeping As it has been suggested in the introductory section, not attacking and waiting has its own value, because additional opportunities that could otherwise not have been foreseen and realized may open up. This value of waiting may indicate that a later conflict could be beneficial. Not attacking also protects rebels from the irreversible costs of an attack. Having the freedom to choose between alternative policies has an extra value that is particularly obvious when talking about violent conflicts. A violent attack lifts the conflict to another level. It removes any opportunity to resolve problems with peaceful measures. With a violent or even deadly attack, such as an assassination of a state representative, there is no turning back; the attackers cannot say, "Sorry, we didn't mean it!". Once the attack is carried out, rebels cannot turn back - they are tied to the expected benefit track they have chosen. This logically corresponds to a firm's investment decision (Dixit and Pindyck, 1994), where the option value of the freedom of choice is a measure of opportunities that may open up in the future when an agent does not irreversibly embark on a particular benefit stream.

Accounting for the option value F_s for the Ito-Lévy Jump Diffusion Process (4.1), we apply dynamic programming to obtain the Hamilton-Jacobi-Bellman equation[51]

$$rF_s dt = E(dF_s).$$

4.2.4 Decision to Attack

The decision to attack straight away is a sequential decision where the rebel repeatedly considers his or her living conditions and evaluates if a conflict at this point in

[51] For a detailed discussion of the option value see Appendix C.2.

time is the best strategy. In order to solve the decision problem of launching the attack, the rebel compares the benefit of immediate conflict V with the option value of a later attack F_s. Therefore, the problem is solved by the solution to

$$\max \{V_s^{gross}(T) - I_s, F_s(T)\}. \tag{4.3}$$

At any time during the non-violent waiting period, the rebel will compare the ENPV of benefit of conflict with the option value of an uncertain non-violent development with the freedom to attack later. If the net value of conflict is greater than the option value ($V_s^{gross}(T) - I_s \geq F_s(T)$), the rebel will carry out the attack. By contrast, if the option value of postponing the attack exists and is greater than the net benefit of attacking straight away, he or she will not initiate the conflict and wait. Solving this continuous sequential decision problem (4.3) also allows us to determine the expected time of the attack.

4.3 Solving for the Expected Time of Conflict

Identifying the conditions that eventually trigger the attack and also determining the expected time of attack involves two steps.

First, for each non-violent period during which deteriorating living conditions generate increasing benefits of the attack, we need to determine the benefit value of conflict in the current period (B^* threshold) that would trigger the outbreak. This threshold is the required current benefit level that would make the attack preferable. It marks the boundary at which conditions become so bad that a conflict becomes unavoidable for the rebel. Then, the expected value of conflict exceeds the option value of peace and hence the attack becomes more profitable. Peaceful waiting, even if uncertain positive events are still possible, is no longer rational.

Second, as the threshold indicates the start of the conflict, rebels simultaneously observe the development of the current period's benefit $\tilde{B}$. Under worsening conditions during the waiting period, they compare the threshold B^* with the corresponding current period's benefit level of conflict $\tilde{B}$ and verify if the threshold has already been reached. Even if the hope for events improving living conditions allow the rebel remain peaceful, the expected end of the peaceful period can be predicted. Hence, understanding this mechanism allows for an extension of peaceful episodes in latent conflicts and may help to generate more time to look for peaceful solutions.

4.3.1 Conflict Threshold

In order to determine the benefit value that triggers the conflict, we need to consider the standard conditions of a stochastic dynamic programming problem. In addition to the Hamilton-Jacobi-Bellman equation for the option value F_s and applying Ito's lemma for jump diffusions to dF_s, we have to use the well-known boundary conditions, namely (4.4), the value matching condition (4.5) and the smooth pasting condition (4.6)

$$F_s(0) = 0, \tag{4.4}$$

$$F_s(B^*) = V_s^{gross}(B^*) - I_s, \tag{4.5}$$

$$\frac{dF_s(B^*)}{dB} = \frac{d(V_s^{gross}(B^*) - I_s)}{dB}, \tag{4.6}$$

to solve for the threshold benefit B^*. The setting of the decision problem implies that the net benefits of conflict must be sufficiently large to launch the attack. Reaching this threshold triggers a change in strategy from peace to conflict. Therefore, determining this threshold is the first part of a solution to the expected timing of attack.

Proposition 16 *Let I_s be the constant costs of conflict,* (4.1) *a sequence of increasing current benefit levels while remaining peaceful, and* (4.2) *future benefit developments after the attack. Further, let β_s be an implicit function resulting from the differential equation $rF_s dt = E(dF_s)$ with solution $F = K_s \tilde{B}^{\beta_s}$ and $K_s \in \mathbb{R}$ being constant. Then the threshold B^* that would trigger a conflict is*

$$B^* = \frac{\beta_s}{\beta_s - 1}(r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) - \int\limits_{U_s} [\ln(1 + z_s) - z_s] \, \upsilon_s(dz_s) - a_s) I_s. \qquad (4.7)$$

Proof. For a proof see Appendix C.3. ■

The threshold is the current benefit that an attack needs to generate as a minimum if all positive values of peaceful waiting are accounted for. It is the ultimate limit to what one can bear in terms of discrimination, oppression, or persecution. As long as this threshold is not reached, the latent conflict is not triggered and the rebel will somehow tolerate the conditions in the hope of improvement. The situation remains calm. However, it is a calm before the storm. Even if the rebel does not attack straight away, his or her expectations about the future suggest that there will be a point in time when the attack is beneficial enough to be launched. That is, knowing the value of the threshold and the random process of living conditions during the peaceful period, the expected time of attack becomes predictable.

4.3.2 Expected Time of Conflict

Once the rebel knows from the threshold at which current benefit level he or she should attack, the question is when he or she can expect to obtain this benefit for the first time. This time is referred to as the first passage time.

As we consider jump diffusion processes, a potential overshooting has to be taken into account. This means that the horizontal boundary marking the potential start of conflict does not have to be hit exactly, but it may be overshot. In our model, we are particularly interested in the effect of repressive actions by the government that are modelled by stochastic events occurring at a random point in time with an unpredictable size. In such a context, we can use the exponential distribution for which an analytical solution for the first passage time exists. Similar to Kou and Wang (2003b) the exponential distribution is given by

$$h_s(z_s) = \eta_s e^{-\eta_s z_s},$$

where $\frac{1}{\eta_s}$ denotes the mean of the exponential distribution. This parameter can be interpreted as the mean waiting time until the next jump occurs. By using the Girsanov theorem, we can derive the probability density function of T^*,[52] which is sometimes referred to as the Inverse Gaussian Distribution.[53]

Proposition 17 *Let $\tilde{B}$ be an Ito-Lévy Jump Diffusion process in (4.1), B^* a constant threshold from (4.7) and $h_s(z_s) = \eta_s e^{-\eta_s z_s}$ be the density function of the double exponential distribution. Then the first hitting time of B^* is*

$$E(T_s^*) = \frac{1}{\bar{u}_s}\left[B^* + \frac{\mu_s^* - \eta_s}{\eta_s \mu_s^*}(1 - e^{-B^* \mu_s^*})\right],$$

with $\bar{u}_s = \tilde{a}_s + \tilde{\lambda}_s \frac{1}{\eta_s}$ denoting the overall drift. μ_s^* is defined as the unique root of $G(\mu_s^*) = 0$ with $G(x) := x\tilde{a}_s + \frac{1}{2}x^2\tilde{\sigma}_s^2 + \tilde{\lambda}_s\left(\frac{\eta_s}{\eta_s - x} - 1\right)$ and $0 < \eta_s < \mu_s^* < \infty$.

Proof. For a proof see Appendix C.3. ■

[52] An extensive discussion is offered by Karatzas and Shreve (1991, p.196), and Karlin and Taylor (1975, p. 363).

[53] The term "Inverse Gaussian Distribution" stems from the inverse relationship between the cumulant generating functions of these distributions and those of the Gaussian distributions. For a detailed discussion of the inverse Gaussian distribution see Johnson et al. (1995) or Dixit (1993).

$E(T_s^*)$ is the result of a sequential optimization and hence the solution to a timing problem. Furthermore, it is the time when the rebel expects to carry out the terrorist attack in order to end repression and discrimination. It is the time that introduces new and improved living conditions. This point in time is just an expected value. In the course of time, a major event can trigger an attack at any, even an unexpected, moment.

4.4 Determinants of the Expected Time of Attack

In this section, we examine how economic conditions or government policies affect stochastic variability and hence the decision of rebels. In general, we could look at both, conditions during the time of a latent conflict which is still non-violent, and changes in expectations about the aftermath of an outbreak. We focus on the conditions during the first, the peace period, where we can already identify a latent conflict but the rebels have not yet turned to violence, and analyse how variability-describing parameters may shorten this peaceful episode.

We start with the effect of usual stochastic economic variations that are described by the volatility of the geometric Brownian motion that as a part of the Ito-Lévy Jump Diffusion process. How does an increase in volatility affect the rebel's decision to attack?

Proposition 18 *An increase in economic variation indicated by the variance of the Brownian motion leads to a later attack*

$$\frac{\partial E(T_s^*)}{\partial \tilde{\sigma}_s} > 0.$$

Proof. For a proof see Appendix C.4. ■

According to Proposition 18, larger variations in the economic, social, or political conditions of the rebel increase the peaceful period. Greater volatility implies both unexpected improved conditions as well as their deterioration. Hence, with this increase in marginal stochastic shocks, the rebel expands the time he or she is willing to hope and wait for the conflict to resolve itself without any violent means.

What happens if there are non-marginal stochastic threats, e.g. if the government steps up the fight against the discriminated group and takes severe oppressive action such as the persecution of group members, waves of detention, or assassination of rebel leaders?

Proposition 19 *An increase in the frequency $\tilde{\lambda}_s$ of large scale repressive government actions during the latent conflict leads to an earlier attack*

$$\frac{\partial E(T_s^*)}{\partial \tilde{\lambda}_s} < 0.$$

Similarly an increase in the magnitude of large scale repressive government actions shortens the peaceful period

$$\frac{\partial E(T_s^*)}{\partial u_s} < 0.$$

Proof. For a proof see Appendix C.4. ■

In general, large non-marginal stochastic shocks, indicated by $\tilde{\lambda}_s$ and u_s, affect the outbreak of conflict. Even more, these effects are different than the effects of marginal shocks, indicated by $\tilde{\sigma}_s$.

An increase in $\tilde{\lambda}_s$ implies that more fundamental events are occurring that imply non-marginal changes in the expected path of benefits associated with the attack. Deteriorations in welfare conditions like unfavourable regime changes, increasing number of oppressive government actions, or even external disasters occur more often. The political implications are straight. In a latent conflict, signals pointing to randomly deteriorating fundamental welfare conditions for rebels would escalate the situation. An earlier attack can be expected. In contrast, a policy that promises significant opportunities for the group represented by the rebels may not terminate the conflict but postpone the attack and give more time for a peaceful conflict resolution.

Furthermore, an increase in u_s means that threats by the government become larger. Hence, a greater benefit of the attack is created. The rebel group faces more severe threats when following the current welfare path. For instance, torture instead of detention, or expropriation instead of taxation during the non-violent period, will make the attack more beneficial since an uprising could lead to a new regime that is more beneficial for the rebels, so we can expect the attack to be carried out sooner.

4.5 Conclusion

There is hardly any decision problem that bears more dramatic and large uncertainties than the decision to launch a violent conflict. In this model the decision to turn to violence is based on the idea that an attack is a form of investment in a change of conditions. The major focus of this paper is on the impact of large (non-marginal) stochastic shocks - such as fundamental threats often created by an oppressive government - on the decision to launch a conflict. In order to capture these major stochastic events, we suggest a real option decision model in the context of social conflicts that predicts the expected outbreak of a conflict. To model such discontinuous large stochastic shocks we introduce a more general stochastic process than the often used geometric Brownian motion, namely an Ito-Lévy Jump Diffusion process. With this stochastic modelling we can show that large stochastic shocks have opposite effects on the decision to launch a conflict than what we usually refer to as risk indicated by the marginal variability, or volatility, included in the geometric

Brownian motion.

In more detail, under dynamic conditions an attack is the result of an evaluation of highly uncertain developments in economic, social, and political conditions. Rebels compare the current conditions with an expected path of development after a potential attack, including conflict costs. If current conditions are expected to develop sufficiently badly, turning violent can be considered and a latent conflict can be identified. During the non-violent period of this latent conflict the benefits of attack are still not sufficiently high, so rebels will remain peaceful and wait until the attack is capable of paying off. The decision to attack is a sequential decision in the course of time that is irreversible. Since we are not interested in the interaction between the government and the rebel but rather in the rebel's reaction on a given but uncertain set of conditions, we suggest that real option theory is an appropriate methodology in this context to evaluate the timing of and make a prediction about a conflict's outbreak. In order to include discontinuities and large stochastic events in our model, we need to extend current methods in real option theory by introducing an Ito-Lévy Jump Diffusion process. For this discontinuous modelling we analytically derive the threshold that triggers the attack, and we determine the time this is expected to happen. Because we propose an "option to attack decision" for formal modelling in conflict theory, we can show that large uncertain shocks have opposite effects than the so far considered volatility. While marginal variability, often modelled by the volatility of a geometric Brownian motion and associated with usual economic or social variation, prolongs the peaceful period, major stochastic shocks, modelled by a jump process and associated with major oppressive government actions, make an early attack more likely. Hence, these large uncertain shocks should be considered more carefully when policy is designed.

The political implication is straight. Even if latent conflicts are not immediately solved, prospects of significant improvements are able to extend the peaceful period and provide more time to find a solution to the conflict. In contrast, more uncertain threats, often meant to awe the oppressed group may provoke an early attack if the threat is strong enough.

Chapter 5

Summary and Conclusion

The recently increased number of social conflicts, natural disasters and economic turbulences has shown that many stochastic events are rare but have a devastating impact on economic activity. Many economic agents suffered from huge losses, which increased the attention of professional interest to develop new evaluation methods of investment projects. As the literature discussion of this thesis emphasizes, for a long time disasters were not regarded as an essential element of investment decisions and only their recent impact changed this practice. The central question that arises in this context is how decision makers can account for large variability. This doctoral thesis sheds further light onto this question by developing methods for evaluating marginal and non-marginal stochastic shocks for non-sequential and sequential investment decisions. Furthermore, it provides an analysis of their effects on investment behaviour as well as statements about optimal investment decisions.

The thesis starts with a brief introduction to the distinction between risk and uncertainty, which was motivated by Knight (1921), and their inclusion into economic decision making. After showing some shortcomings of the evaluation methods provided by the literature, the aim of this thesis is to include both types of stochastic variability, namely marginal and non-marginal, and to show their ambiguous effects on investment decisions. Hence, the core of this thesis consists of three research papers that contribute to approaching this goal economically and mathematically.

Methodologically, the three papers are built on each other by extending and generalizing the previous evaluation method.

The first paper, *Investment under Threat of Disaster,* introduces an extension of the Expected Net Present Value, which is the simplest and most often used dynamic evaluation method. At the same time, the ENPV builds the starting point of this thesis. In particular, the Ito-Lévy Jump Diffusion process, which is a geometric Brownian motion with accumulated random jumps, is implemented and compared to the workhorse stochastic process, the geometric Brownian motion. Even with this simple method it can be shown that disastrous events strongly affect current conditions so that they may even overthrow them. It is confirmed that the ENPV does not account for marginal variability but it does for non-marginal variability. An additional element in the discount factor that summarizes the effects of non-marginal stochastic shocks on the expected project value is obtained. For an investment project, this additional parameter may cause an apparently beneficial project to become worthless. Hence, disregarding potential disasters leads to an overestimation of projects. From this analysis a conclusion can be formed: Simple probabilistic statements, in particular when considering large or even "overthrowing" events, are misleading and an insufficient description of the degree of randomness. With the developed method it is not necessary to state the probability that a certain disaster will happen during a specified future time period and we can still evaluate the rather uncertain future using the simple ENPV.

The second paper, *Stay in school or start working? - The Human Capital Investment Decision Under Uncertainty and Irreversibility*, uses the previously developed ENPV and implements it to a sequential real option model in the frame of a human capital investment decision. Two periods are considered, namely the pure investment phase, which improves the outcome of the project, and the stochastic earning phase.

The investor does not know how long he or she has to invest and how much he or she will have to accumulate in order to maximize the education value. The decision is based on a real option model that accounts for accumulated education costs, the complete earnings profiles including entry-level wage, sheepskin effects and earning dynamics, as well as the option value of schooling introducing potential career opportunities or threats. Hence, in this paper, uncertainty again is a major component of the schooling decision, so that an Ito-Lévy Jump Diffusion process is used to model marginal and non-marginal stochastic shocks during the career. The model provides a solution to the problem of minimal compensation that schooling needs to generate as well as the optimal time to leave school. Furthermore, the analysis shows that an increase in education costs causes the student stay in the system as long as the increasing costs of schooling are sufficiently compensated by the market. Sheepskin effects may produce such an extra income premium. Staying at school becomes less attractive if the frequency of opportunities offered by the market occur more often than threats.

The last paper, *Uncertainty and Conflict Decision*, extends the previous one by also allowing for non-marginal stochastic shocks during the waiting period. With this approach, the effect of such stochastic events on the outbreak of conflict, hence, an investment into a change of the status quo, is analysed. The decision is sequential and irreversible in nature so that a real option model is applied. Discontinuous large stochastic shocks, again modelled by Ito-Lévy Jump Diffusion processes, may be generated by the government so that they may cause or prevent the outbreak. With this stochastic modelling, the threshold that triggers the attack and the time when this is expected to happen is determined. Furthermore, it can be shown that uncertain shocks have opposite effects on the decision to launch a conflict than what we usually refer to as risk indicated by the marginal variability, or volatility, included in the geometric Brownian motion. In more detail, marginal variability associated with usual economic or social variation, prolongs the peaceful period while major uncertain shocks associated with major oppressive government actions, make an early attack more likely.

To sum up, this thesis shows that for investment decisions it is important to consider not only marginal variability but also non-marginal stochastic shocks. Both can have opposed effects on investment projects. That is, marginal variability, which is usually depicted by the volatility of the geometric Brownian motion, may not affect the decision of the investor if the ENPV rule is used. If instead, the investment decision is determined sequentially, then the project value may be affected positively by marginal variation. The contrary result, however, is obtained if the investor includes the possibility of large uncertain events, such as non-marginal stochastic shocks that occur at an uncertain point in time with an uncertain impact. These large shocks may be so severe that they may make an initially profitable project to become worthless. Although this thesis provides three different approaches for the evaluation of stochastic variability, especially of non-marginal stochastic shocks, it can only be seen as a starting point for a new research field. This thesis shows how important it is to account for both types of variability in economic decision making and that disregarding either type of variability may result in non-profitable economic decisions. For this reason, there is a need to reconsider the general view of the evaluation of risk and uncertainty. One can begin with the classical Expected Utility Approach in order to show how large uncertainty enters the utility evaluation of agents and then change over to more sophisticated techniques.

References

Abundo, M. (2010): On the First Hitting Time of an One-Dimensional Diffusion and a Compound Poisson Process. Methodology and Computing in Applied Probability, Vol. 12, pp. 473-490.

Alexander, C.; Kaeck, A. (2012): Does Model Fit Matter for Hedging? Evidence from FTSE 100 Options. Journal of Futures Markets, Vol. 32 (7), pp. 609-638.

Altonji, J. G. (1993): The Demand for and Return to Education when Education Outcomes are Uncertain. Journal of labour Economics, Vol. 11 (1), pp. 48-83.

Amin, K. I. (1993): Jump Diffusion Option Valuation in Discrete Time. The Journal of Finance, Vol. 48 (5), pp. 1833-1863.

Anderton, C. H.; Carter, J. R. (2005): On Rational Choice Theory and the Study of Terrorism. Defence and Peace Economics, Vol. 16 (4), pp. 275-282.

Andersen, T. G.; Davis, R. A.; Kreiß, J. P.; Mikosch, T.; Eds., Eberlein, E. (2009): Handbook of Financial Time Series. Springer, pp. 439-455.

Applebaum, D. (2009): Lévy Processes and Stochastic Calculus. Cambridge University Press, 2nd edition, Cambridge, UK.

Arrow, K. J. (1971): Essays in the Theory of Risk-Bearing. The Journal of Business, Chicago: Markham Publishing Co.

Barro, R. J. (2006): Rare Disasters and Asset Markets in the Twentieth Century. The Quarterly Journal of Economics, August 2006, pp. 823-866.

Barro, R. J.; Ursua, J. F. (2008): Consumption Disasters in the Twentieth Century. The American Economic Review, Vol. 98 (2), pp. 58-63.

Bayraktar, E.; Xing, H. (2009): Analysis of the Optimal Exercise Boundary of American Options for Jump Diffusions. Siam Journal on Mathematical Analysis, Vol. 41 (2), pp. 825-860.

Belzil, C.; Leonardi, M. (2007a): Can Risk Aversion Explain Schooling Attainments? Evidence from Italy. Labour Economics, Vol. 14 (6), pp. 957-70.

Belzil, C.; Leonardi, M. (2007b): Risk Aversion and Schooling Decisions. IZA Discussion Paper No. 2994.

Berrebi, C.; Ostwald, J. (2011): Earthquakes, Hurricanes, and Terrorism: Do Natural Disasters incite Terror?. Public Choice, Vol. 149, pp. 383-403.

Besley, T.; Persson, T. (2011): The Logic of Political Violence. The Quarterly Journal of Economics, Vol. 126, pp. 1411-1445.

Bilkic, N.; Gries, T.; Pilichowski, M. (2012): Stay in School or Start Working?- The Human Capital Investment Decision Under Uncertainty and Irreversibility. Labour Economics, Vol. 19 (5), pp. 706-717.

Black, F.; Scholes, M. (1973): The Pricing of Options and Corporate Liabilities. Journal of Political Economy, Vol. 81 (3), pp. 637-654.

Blattmann, C.; Miguel, E. (2010): Civil War. Journal of Economic Literature, Vol. 48 (1), pp. 3–57.

Blomberg, S. B.; Hess, G. D.; Weerapana, A. (2004): Economic Conditions and Terrorism. European Journal of Political Economy, Vol. 20, pp. 463-478.

Born, P.; Viscusi, W. K. (2006): The Catastrophic Effects of Natural Disasters on Insurance Markets. Journal of Risk and Uncertainty, Vol. 33, pp. 55-72.

Brancati, D. M. (2007): Political Aftershocks: The Impact of Earthquakes on Intrastate Conflict. Journal of Conflict Resolution, Vol. 51 (5), pp. 715-743.

Cai, N.; Kou, S. G. (2011): Option Pricing Under a Mixed-Exponential Jump Diffusion Model, Management Science, Vol. 57 (11), pp. 2067–2081.

Caplan, B. (2006): Terrorism: The Relevance of the Rational Choice Model. Public Choice, Vol. 128 (1), pp. 91-107.

Carneiro, P.; Hansen, K.; Heckman, J. J. (2003): Estimating Distributions of Treatment Effects with an Application to the Returns to Schooling and Measurement of the Effects of Uncertainty on College Choice. International Economic Review, Vol. 44 (2), pp. 361–422.

Cartea, A.; Meyer-Brandis, T. (2010): How Duration Between Trades of Underlying Securities Affects Option Prices. Review of Finance, Vol. 14, pp. 749–785.

Chow, G.; Kritzman, M. (2002): Value at Risk for Portfolios with Short Positions: Do Not Assume Lognormal Distributions. Journal of Portfolio Management, Vol. 28 (3), pp. 73-81.

Collier, P.; Höffler, A. (1998): On Economic Causes of Civil War. Oxford Economic Papers, Vol. 50, pp. 563-573.

Comay, Y.; Melnik, A.; Pollatschek, M. A. (1973): The Option Value of Education and the Optimal Path for Investment in Human Capital. International Economic Review, Vol. 14 (2), pp. 421-435.

Cont, R.; Eberlein, E. (2010): Encyclopedia of Quantitative Finance. John Wiley & Sons Ltd., pp. 990-994.

Cont, R.; Tankov, P. (2004): Financial Modelling With Jump Processes. Chapman & Hall/CRC Financial Mathematics Series.

Cox, S. H.; Fairchild, J.; Pedersen, H. (2004): Valuation of Structured Risk Management Products. Insurance: Mathematics and Economics, Vol. 34, pp. 259-272.

Cox, S. H.; Pedersen, H. (2000): Catastrophe Risk Bonds. North American Actuarial Journal, Vol. 4 (4), pp. 56-82.

Cox, J. C.; Ross, S. A. (1976): The Valuation of Options for Alternative Stochastic Processes. Journal of Financial Economics, Vol. 3, pp. 145-166.

Cox, J. C.; Ross, S. A.; Rubinstein, M. (1979): Option Pricing: A Simplified Approach. Journal of Financial Economics, Vol. 7, pp. 229-263.

Cunha, F.; Heckman, J. J.; Navarro, S. (2005): Separating Uncertainty from Heterogeneity in Life Cycle Earnings. The 2004 Hicks Lecture. Oxford Economic Papers, Vol. 57 (2), pp. 191-261.

Denny, K.; Harmon, C. (2001): Testing for Sheepskin Effects in Earnings Equations: Evidence for Five Countries. Applied Economic Letters, Vol. 8, pp. 635-637.

Dixit, A. K. (1993): The Art of Smooth Pasting. Chur, Switzerland: Harwood Academic Publishers.

Dixit, A. K.; Pindyck, R. S. (1994): Investment Under Uncertainty. Princeton Press.

Eaton, J.; Rosen, H. (1980): Taxation, Human Capital and Uncertainty. The American Economic Review, Vol. 70 (4), pp. 705-715.

Fabozzi, F. J.; Kolm, P.; Pachamanova, D. A.; Focardi, S. M. (2007): Robust Portfolio Optimization and Management. New Jersey: John Wiley&Sons, Inc.

Fabozzi, F. J.; Markowitz, H. (2002): The Theory and Practice of Investment Management. New Jersey: John Wiley&Sons, Inc.

Fearon, J. D. (1995): Rationalist Explanation of War. International Organization, Vol. 49 (3), pp. 379-414.

Ferrer, A.; Riddell, W. C. (2008): Education, Credentials, and Immigrant Earnings. Canadian Journal of Economics, Vol. 41 (1), pp. 186-216.

Finkel, S. E.; Muller, E. N. (1998): Rational Choice and the Dynamics of Collective Political Action. Evaluating Alternative Models with Panel Data. The American Political Science Review, Vol. 92 (1), pp. 37-49.

Gabaix, X. (2008): Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-Finance. The American Economic Review, Vol. 98, (2), pp. 64-67.

Gabaix, X. (2012): Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance. The Quarterly Journal of Economics, Vol. 127, pp. 645–700.

Gould, E. D.; Klor, E. F. (2010): Does Terrorism Work? The Quarterly Journal of Economics, Nov. 2010, pp. 1459-1510.

Gourio, F. (2012): Disaster Risk and Business Cycles. American Economic Review, Vol. 102 (6), pp. 2734–2766.

Graham, D. (1981): Cost-Benefit Analysis under Uncertainty. The American Economic Review, Vol. 41 (4), pp. 715-725.

Gries, T.; Meierrieks, D. (2013): Do Banking Crisis cause Terrorism? Economics Letters, Vol. 119 (3), pp. 321-324.

Groot, W.; Oosterbreek, H. (1992): Optimal Investment in Human Capital Under Uncertainty. Economics of Education Review, Vol. 11 (1), pp. 41-49.

Grossman, H. I. (1991): A General Model of Insurrections. The American Economic Review, Vol. 81 (4), pp. 912-921.

Hadar, J.; Russels, W. R. (1969): Rules for Ordering Uncertain Prospects. The American Economic Review, Vol. 59 (1), pp. 25-34.

Hanchane, S.; Lioui, A.; Touahri, D. (2006): Human Capital as a Risky Asset and the Effect of Uncertainty on the Decision to Invest. LEST Working Paper.

Hartog, J.; Van Ophem, H.; Bajdechi, S. M. (2007): Simulating the Risk of Investment in Human Capital. Education Economics, Vol. 15 (3), pp. 259–275.

Heckman, J. J.; Lochner, L. J.; Todd, P. E. (2003): Fifty Years of Mincer Earnings Regressions. Technical Report 9732, National Bureau of Economic Research.

Heckman, J. J.; Lochner, L. J.; Todd, P. E. (2006): Earnings Equations and Rates of Return: The Mincer Equation and Beyond. E. A. Hanushek and F. Welch (Eds.), Handbook of the Economics of Education, Chapter 7, pp. 307-458.

Heckman, J. J.; Lochner, L.; Todd, P. E. (2008): Earning Functions and Rates of Return. Journal of Human Capital, Vol. 2 (1), pp. 1-128.

Hogan, V.; Walker, I. (2007): Education Choice under Uncertainty: Implications for Public Policy. Labour Economics, Vol. 14, pp. 894-912.

Jacobs, B. (2007): Real Options and Human Capital Investment. Labour Economics, Vol. 14, pp. 913-925.

Jarrow, R.; Zhao, F. (2006): Downside Loss Aversion and Portfolio Management. Management Science, Vol. 52 (4), pp. 558–566.

Jang, J. (2007): Jump Diffusion Processes and their Applications in Insurance and Finance. Insurance: Mathematics and Economics, Vol. 41, pp. 62-70.

Johnson, N. L.; Kotz, S.; Balakrishnan, N. (1995): Continuous Univariate Distributions. 2nd edition, New York: John Wiley & Sons, Inc.

Karatzas, I.; Shreve, S. E. (1991): Brownian Motion and Stochastic Calculus, 2nd edition, Springer.

Karlin, S.; Taylor, H. M. (1975): A First Course in Stochastic Processes. Academic Press.

Katz, L. F.; Autor, D. H. (1999): Changes in the Wage Structure and Earnings Inequality. O. Ashenfelter and D. Card (Eds.), Handbook of labour Economics, Vol. 3.

Knight, F. H. (1921): Risk, Uncertainty, and Profit. Boston, MA: Hart, Schaffner & Marx; Houghton Mifflin Company.

Kou, S. G. (2002): A Jump-Diffusion Model for Option Pricing. Management Science, Vol. 48 (8), pp. 1086-1101.

Kou, S. G.; Wang, H. (2003a): Option Pricing under a Double Exponential Jump Diffusion Model. Management Science, Vol. 50 (9), pp. 1178-1192.

Kou, S. G.; Wang, H. (2003b): First Passage Time of a Jump Diffusion Process. Advances in Applied Probability, Vol. 35 (2), pp. 504-531.

Kousky, C. (2010): Learning from Extreme Events: Risk Perceptions after the Flood. Land Economics, University of Wisconsin Press, Vol. 86 (3), pp. 395-422.

Kruschwitz, L. (2011): Investitionsrechnung. 13th edition, Oldenburg Verlag München.

Kunreuther, H. (1996): Mitigating Disaster Losses Through Insurance. Journal of Risk and Uncertainty, Vol. 12, pp. 171-187.

Kunreuther, H.; Kleffner, A. (1992): Should Earthquake Mitigation Measures be Voluntary or required? Journal of Regulatory Economics, Vol. 4, pp. 321–335.

Kunreuther, H.; Novemsky, N.; Kahneman, D. (2001): Making Low Probabilities useful. Journal of Risk and Uncertainty, Vol. 23 (2), pp. 103–120.

Kunreuther, H.; Pauly, M. (2004): Neglecting Disaster: Why Don't People Insure Against Large Losses? Journal of Risk and Uncertainty, Vol. 28 (1), pp. 5–21.

Lave, L. B.; Apt, J. (2006): Planning for Natural Disasters in a Stochastic World. Journal of Risk and Uncertainty, Vol. 33, pp. 117–130.

Layard, R.; Glaister, S. (1994): Cost-Benefit Analysis. Cambridge University Press, Cambridge, UK.

Levhari, D.; Weiss, Y. (1974): The Effect of Risk on the Investment in Human Capital. The American Economic Review, Vol. 64 (6), pp. 950-963.

Markowitz, H. (1952): Portfolio Selection. The Journal of Finance, Vol. 7 (1), pp. 77-91.

Mehra, R.; Prescott, E. C. (1985): The Equity Risk Premium: A Puzzle. Journal of Monetary Economics, Vol. 15, pp. 145-161.

Merton, R. C. (1975): Option Pricing when the Stock Returns are Discontinuous. Journal of Financial Economics, Vol. 3, pp. 125-144.

Mincer, J. (1974): Schooling, Experience and Earnings. New York: National Bureau of Economic Research.

Morrow, J. D. (1985): A Continuous-Outcome Expected Utility Theory of War. The Journal of Conflict Resolution, Vol. 29 (3), pp. 473-502.

Muller, E. N.; Opp, K. D. (1986): Rational Choice and Rebellious Collective Action. The American Political Science Review, Vol. 80 (29), pp. 471-488.

Oksendal, B. (2004): Stochastic Differential Equations: An Introduction with Applications. 6th edition, Springer.

Oksendal, B.; Sulem, A. (2007): Applied Stochastic Control of Jump Diffusions. 2nd edition, Springer.

Powell, R. (2002): Bargaining Theory and International Conflict. Annual Review of Political Science, Vol. 5, pp. 1–30.

Powell, R. (2006): War as a Commitment Problem. International Organization 60, Winter 2006, pp. 169-203.

Protter, P. (1990): Stochastic Integration and Differential Equation: A New Approach. Springer.

Pham, H. (1997): Optimal Stopping, Free Boundary, and American Option in a Jump-Diffusion Model. Applied Mathematics and Optimization, Vol. 35, pp. 145-164.

Pratt, J. W. (1966): Risk Aversion in the Small and in the Large. Econometrica, Vol. 32 (1/2), pp. 122-136.

Prest, A. R.; Turvey, R. (1965): Cost-Benefit Analysis: A Survey. The Economic Journal, Vol. 75, pp. 683-735.

Rietz, T. A. (1988): The Equity Risk Premium: A Solution. Journai of Monetary Economics, Vol. 22, pp. 117-131.

Roy, A. D. (2001): Safety First and the Holding of Assets. Behavioural Finance, Vol. 3, pp. 11-29.

Sandler, T.; Enders, W. (2004): An Economic Perspective on Transnational Terrorism. European Journal of Political Economy, Vol. 20, pp. 301-316.

Sassone, P. G.; Schaffer, W. A. (1978): Cost-Benefit Analysis - A Handbook. Academic Press, Inc, San Diego, US.

Silles, M. (2008): Sheepskin Effects in the Returns to Education. Applied Economics Letters, Vol. 15 (3), pp. 217-219.

Skalli, A. (2007): Are Successive Investments in Education Equally Worthwhile? Endogenous Schooling Decisions and Non-Linearities in the Earnings-Schooling Relationship. Economics of Education Review, Vol. 26, pp. 215-231.

Skaperdas, S. (1992): Cooperation, Conflict, and Power in the Absence of Property Rights. The American Economic Review, Vol. 82 (4), pp. 720-739.

Smith, V. K.; Carbone, J. C.; Pope, J. C.; Hallstrom, D. G.; Darden, M. E. (2006): Adjusting to Natural Disasters. Journal of Risk and Uncertainty, Vol. 33, pp. 37–54.

Sutter, D.; Poitras, M. (2010): Do People Respond to Low Probability Risks? Evidence from Tornado Risk and Manufactured Homes. Journal of Risk and Uncertainty, Vol. 40, pp. 181–196.

Tornell, A. (1998): Reform from Within. NBER Working Paper No. 6497.

Trigeorgis, L. (1996): Real Options: Managerial Flexibility and Strategy in Resource Allocation. Cambridge, MA: MIT Press.

Unser, M. (2002): Lower Partial Moments as Measures of Perceived Risk: An Experimental Study. Journal of Economic Psychology, Vol. 21 (3), pp. 253-280.

Victoroff, J. (2005): The Mind of the Terrorist: A Review and Critique of Psychological Approaches. The Journal of Conflict Resolution, Vol. 49 (1), pp. 3-42.

Viscusi, W. K. (2009). Valuing Risks of Death from Terrorism and Natural Disasters. Journal of Risk and Uncertainty, Vol. 38, pp. 191 –213.

Wachter, J. A. (2013): Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? Journal of Finance, Vol. LXVIII (3), pp. 987-1035.

Wang, Y.; Bai, C. (2003): Uncertainty in labour Productivity and Specific Human Capital Investment. Journal of labour Economics, Vol. 21 (3), pp. 651-675.

Weede, E.; Muller, E. N. (1998): Rebellion, Violence and Revolution: A Rational Choice Perspective. Journal of Peace Research, Vol. 35 (1), pp. 43-59.

Weisbrod, B. A. (1962): Education and Investment in Human Capital. Journal of Political Economy, Vol. 70 (5), pp. 106–123.

Williams, J. (1978): Risk, Human capital, and the Investor's Portfolio. Journal of Business, Vol. 55, pp. 65-89.

Yang, H.; Zhang, L. (2005): Optimal Investment for Insurers with Jump-Diffusion Risk Process. Insurance: Mathematics and Economics, Vol. 37, pp. 615-634.

Yared, P. (2010): A Dynamic Theory of War and Peace. Journal of Economic Theory, Vol. 145, pp. 1921-1950.

Zeckhauser, R. (1996): The Economics of Catastrophes. Journal of Risk and Uncertainty, Vol. 12, pp. 113-140.

Appendix A

Investment under Threat of Disaster

A.1 Geometric Brownian Motion

Let $\tilde{P}(t)$ be a geometric Brownian motion defined by the following stochastic differential equation

$$d\tilde{P}(t) = \tilde{a}_p\tilde{P}(t)dt + \tilde{\sigma}_p\tilde{P}(t)d\tilde{W}_p, \text{ with } \tilde{P}(0) = \tilde{p}_0,$$

where $\tilde{a}_p$, $\tilde{\sigma}_p \in \mathbb{R}_+$ are non-negative constants. $d\tilde{W}_p$ describes the increment of the standard Wiener Process. From Dixit and Pindyck (1994) and Oksendal (2004) we know that it has the expected value

$$E\tilde{P}(t) = E\tilde{P}(0)e^{\tilde{a}_p t}.$$

Proof of Proposition 1. The Expected Net Present Value of the future benefit stream $\tilde{P}$ is

$$\begin{aligned} ENPV_1 &= -I_p + E\left(\int_T^{\infty} \tilde{P}(t) e^{-r(t-T)} dt\right) \\ &= -I_p + \left[\frac{1}{\tilde{a}_p - r} e^{-r(t-T)} e^{\tilde{\alpha}_p t} \tilde{P}(t)\right]_T^{\infty} \\ &= -I_p + \frac{\tilde{P}(T)}{r - \tilde{\alpha}_p}; \quad r > \tilde{a}_p. \end{aligned}$$

■

A.2 Ito-Lévy Jump Diffusion Process

Let P be an Ito-Lévy Jump Diffusion Process which is defined by the following stochastic differential equation

$$dP = P a_p dt + P \sigma_P dW_P + P \int_{U_P} z_p N_p(t, dz_P), \; P(0) = p_0,$$

with a_p, $\sigma_P \in \mathbb{R}_+$ and constant. dW_P denotes the increment of the Wiener process and N_p stands for the Poisson process with intensity λ_p. A more general formulation of this process can be found in Oksendal and Sulem (2007). They describe under which conditions a solution to this SDE exists and discuss some characteristics. For our purpose we assume that the existence conditions are fulfilled. A further discussion of Lévy processes and their characteristics can be found in, e.g. Applebaum (2009) and in Cont and Tankov (2004).

A.2.1 Solution to the SDE

Similar to the geometric Brownian motion, define $X_p(t) := \ln P(t)$ and use Ito's Lemma for jump processes in order to find the solution to the SDE.[54] The solution is

$$P(t) = P(0)\exp\left[\begin{array}{c} \left(a_p - \frac{1}{2}\sigma_P^2\right)t + \sigma_P W_p(t) + \int_0^t \int_{U_P} \left[\ln(1+z_p) - z_p\right]\upsilon_p(dz_p)ds \\ + \int_0^t \int_{U_P} \ln(1+z_p) N_p(ds, dz_p) \end{array}\right].$$

Note that the function $P(t)$ is only defined for $z_p > -1$.

A.2.2 Expected Value

Proof of Proposition 2. Since the Wiener and the Poisson processes are independent, we can obtain the expected value of P by separating the components

$$\begin{aligned} EP(t) &= \underbrace{P(0) \cdot Ee^{\left(a_p - \frac{1}{2}\sigma_P^2\right)t + \sigma_P W_p(t)}}_{(1)} \cdot \underbrace{E\left[\exp\left(\int_0^t \int_{U_p} \ln(1+z_p) N_p(ds, dz_p)\right)\right]}_{(2)} \\ &\quad \underbrace{\cdot E\left[\exp\left(\int_0^t \int_{U_P} \left[\ln(1+z_p) - z_p\right]\upsilon_p(dz_p)ds\right)\right]}_{(3)}. \end{aligned}$$

We know that the expected value of (1) is the same as for the geometric Brownian motion $P(0)e^{a_p t}$. For (2) we use that

$$\int_0^t \int_{U_p} \ln(1+z_p) N_p(ds, dz_p)$$

---

[54] See also Oksendal and Sulem (2007).

is compound Poisson distributed and has the characteristic function[55]

$$E\left[\exp\left(iu_p\int_0^t\int_{U_p}\ln(1+z_p)N_p(ds,dz_p)\right)\right]=\exp\left(t\int_{U_p}\left(e^{iu_pz_p}-1\right)\upsilon_{pf}(dz_p)\right),$$

with $\upsilon_{pf}=\upsilon_p\circ f_p^{-1}$, $f_p=\ln(1+z_p)$ and $u_p\in\mathbb{C}$ constant. By choosing $u_p=-i$ we obtain

$$\begin{aligned}
E\left[\exp\left(\int_0^t\int_{U_p}\ln(1+z_p)N_p(ds,dz_p)\right)\right] &= \exp\left(t\int_{U_p}\left(e^{z_p}-1\right)\upsilon_{pf}(dz_p)\right)\\
&= \exp\left(t\int_{U_p}\left(e^{z_p}-1\right)\left(\upsilon(dz_p)\circ f_p^{-1}\right)\right)\\
&= \exp\left(t\int_{f_p^{-1}(U_p)}\left[\left(e^{z_p}-1\right)\circ\ln(1+z_p)\right]\upsilon_p(dz_p)\right)\\
&= \exp\left(t\int_{f_p^{-1}(U_p)}z_p\upsilon_p(dz_p)\right).
\end{aligned}$$

In order to ensure the existence of the expected value, we have to assume that all moments are finite. For (3) we obtain

$$\begin{aligned}
E\left[\exp\left(\int_0^t\int_{U_P}\ln(1+z_p)-z_p\upsilon_p(dz_p)ds\right)\right] &= E\left[\exp\left(t\int_{U_p}\ln(1+z_p)-z_p\upsilon_p(dz_p)\right)\right]\\
&= \exp\left(t\int_{U_p}\ln(1+z_p)-z_p\upsilon_p(dz_p)\right).
\end{aligned}$$

[55]See Theorem 2.3.7 (i) in Applebaum (2009).

Therefore, the expected value of P is

$$\begin{aligned} EP(t) &= P(0)e^{a_p t}\cdot\exp\left(t\int\limits_{f_p^{-1}(U_p)} z_p\upsilon_p(dz_p)\right)\cdot\exp\left(t\int\limits_{U_p}\ln(1+z_p)-z_p\upsilon_p(dz_p)\right) \\ &= P(0)\exp\left[t\left(a_p+\int\limits_{f_p^{-1}(U_p)} z_p\upsilon_p(dz_p)+\int\limits_{U_p}\ln(1+z_p)-z_p\upsilon_p(dz_p)\right)\right] \\ &= P(0)\exp\left[t\left(a_p+\lambda_p\int\limits_{f_p^{-1}(U_p)} z_p h_p(dz_p)+\lambda_p\int\limits_{U_p}\ln(1+z_p)-z_p h_p(dz_p)\right)\right], \end{aligned}$$

with h_p being the distribution of jump sizes. ■

Proof of Proposition 3. The Expected Net Present Value using P is determined as

$$\begin{aligned} ENPV_2 &= -I_p+E\int\limits_T^\infty e^{-r(t-T)}P(t)dt \\ &= -I_p+\int\limits_T^\infty e^{-r(t-T)}P(T)\exp\left[(t-T)\left(\begin{array}{c} a_p+\int\limits_{f_p^{-1}(U_p)} z_p\upsilon_p(dz_p) \\ +\int\limits_{U_p}\ln(1+z_p)-z_p\upsilon_p(dz_p)\end{array}\right)\right]dt \\ &= -I_p+\frac{P(T)}{\left(r-\lambda_p\int\limits_{f_p^{-1}(U_p)} z_p h_p(dz_p)-\lambda_p\int\limits_{U_p}\ln(1+z_p)-z_p h_p(dz_p)-a_p\right)}, \end{aligned}$$

with

$$r>\lambda_p\int\limits_{f_p^{-1}(U_p)} z_p h_p(dz_p)+\lambda_p\int\limits_{U_p}\ln(1+z_p)-z_p h_p(dz_p)+a_p.$$

This condition is necessary because otherwise the ENPV would be infinite. ■

A.3 Effects of Disasters on the ENPV

The derivative of $ENPV_2$ with respect to σ_p is

$$\frac{\partial ENPV_2}{\partial \sigma_p} = 0.$$

The derivative of $ENPV_2$ with respect to λ_p is

$$\frac{\partial ENPV_2}{\partial \lambda_p} = -\delta \frac{P(T)}{(r - \lambda_p \delta - a_p)^2} < 0.$$

$I_0[mio]$	$P(T)[mio]$	r	a_p	m_p	$1-m_p$	η_{p_1}	η_{p_2}	λ_p	dis. int	opport. int	$ENPV_2$
1	0,1	0,05	0,02	0	1	∞	2	0,00	−0,00	0	2,33
1	0,1	0,05	0,02	0	1	∞	2	0,05	−0,05	0	2,31
1	0,1	0,05	0,02	0	1	∞	2	0,10	−0,10	0	2,17
1	0,1	0,05	0,02	0	1	∞	2	0,20	−0,20	0	1,49
1	0,1	0,05	0,02	0	1	∞	2	0,40	−0,40	0	0,11
1	0,1	0,05	0,02	0	1	∞	2	0,80	−0,80	0	−0,68
1	0,1	0,05	0,02	0	1	∞	2	1,60	−1,60	0	−0,90
1	0,1	0,05	0,02	0	1	∞	2	3,20	−3,20	0	−0,96
1	0,1	0,05	0,02	0	1	∞	2	6,40	−6,40	0	−0,98

Table A.1: Example for the Effects of Disasters on Project Value

Appendix B

Stay in School or Start Working? - The Human capital Investment Decision Under Uncertainty and Irreversibility

B.1 Solution and Expected Value of the SDE

In order to find the solution of the geometric Brownian motion $\tilde{Y}$, define $\tilde{X}(t) = \ln \tilde{Y}(t)$ and use Ito's Lemma. In line with Oksendal (2004) the solution to the SDE is $\tilde{Y}(t) = \tilde{y}_0 e^{((\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)t + \tilde{\sigma}_h \tilde{W}_h)}$. The expected value can be found in Dixit and Pindyck (1994) which is $E\tilde{Y}(t) = \tilde{Y}(0)e^{\tilde{\alpha}_h t}$. For the solution of the SDE for Y, replace $\tilde{Y}$ by Y.

B.2 Decision Components

B.2.1 Expected Net Present Value of the Earning Streams

The value of the earnings stream is determined by

$$V_h^{gross} = E\int_T^{\infty} e^{-r(t-T)} Y(t)dt = \frac{Y(T)}{r - a_h}$$

for $r > a_h$.

For the net value of the earnings stream, reduce V_h^{gross} by the education costs.

B.2.2 Option Value of Further Education

For the option value F_h, the Hamilton-Jacobi-Bellman equation in Dixit and Pindyck (1994) holds. Using Ito's Lemma and $E(d\tilde{W}_h) = 0$ we obtain a second-order homogenous ordinary differential equation with a free boundary which has the general solution $F = K_h \tilde{Y}^{\beta_h}$. After inserting this solution, we obtain

$$\beta_h = \frac{1}{2} - \frac{\tilde{a}_h}{\tilde{\sigma}_h^2} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{a}_h}{\tilde{\sigma}_h^2}\right)^2 + \frac{2r}{\tilde{\sigma}_h^2}} > 1 \qquad \text{see (3.9)}$$

For the derivative of β_h with respect to $\tilde{\sigma}_h$ we obtain

$$\frac{d\beta_h}{d\tilde{\sigma}_h} = \frac{2\tilde{a}_h \left[\left(\frac{1}{2} - \frac{\tilde{a}_h}{\tilde{\sigma}_h^2}\right)^2 + \frac{2r}{\tilde{\sigma}_h^2}\right]^{-\frac{1}{2}}}{\tilde{\sigma}_h^3} \left[\left[\left(\frac{1}{2} - \frac{\tilde{a}_h}{\tilde{\sigma}_h^2}\right)^2 + \frac{2r}{\tilde{\sigma}_h^2}\right]^{\frac{1}{2}} + \frac{1}{2} - \frac{\tilde{a}_h}{\tilde{\sigma}_h^2} - \frac{r}{\tilde{\sigma}_h}\right] < 0.$$

B.2.3 Entry Threshold

Proof of Proposition 4. According to the value matching condition (3.6), at the investment trigger point Y^* the value of the option must equal the net value obtained by exercising it (value of the active project minus sunk cost of the investment). The smooth-pasting condition (3.7) requires that the two value functions meet tangentially. Both conditions can be found in Dixit and Pindyck (1994). Hence, by using these two conditions, the threshold Y^* is

$$Y^*(T) = \frac{\beta_h}{\beta_h - 1}(r - a_h)I_h(T).$$

■

B.3 Deriving the Expected First Time Realization of Entry-Level Wages

B.3.1 Expected Time before Market Entry

Proof of Proposition 5. We can determine the expected time $E(\tilde{T}_i)$ needed to reach a certain income level $\tilde{y}$ for the first time given the present value $\tilde{y}_0$. By using the Girsanov theorem, we derive the probability density function of $\tilde{T}_h$.[56] With the Laplace transform[57] of $\tilde{T}_h$ we determine the expected time before market entry, which is

$$E(\tilde{T}_h) = \frac{\ln(\frac{\tilde{y}}{\tilde{y}_0})}{\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2}.$$

For each $\tilde{y}_h$ we can determine each expected time $E(\tilde{T}_h)$ when this entry-level wage is reached for the first time. Hence, for a continuous variation of $\tilde{y} > \tilde{y}_0, \tilde{y} \in \mathbb{R}$ we can write $E(\tilde{T}_i)$ as a function of any potential entry-level wage $\tilde{y}$. Later we discuss the existence of the expected time T^* of market entry for the threshold $Y^*(T)$ (first passage time for the threshold Y^* (see 3.8)). Therefore, for each existing $E\tilde{T}_h = T$, we rewrite (3.10) as a continuous function f of time T

$$\tilde{y}_0 e^{T(\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)} =: f(T).$$

$\ln f(T)$ is a linear function in T

$$\ln f(T) = \ln \tilde{y}_0 + T(\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2).$$

■

[56] An extensive discussion is offered by Karatzas and Shreve (1991, p.196), and by Karlin and Taylor (1975, p.363).
[57] See Ross (1996), Proposition 8.4.1.

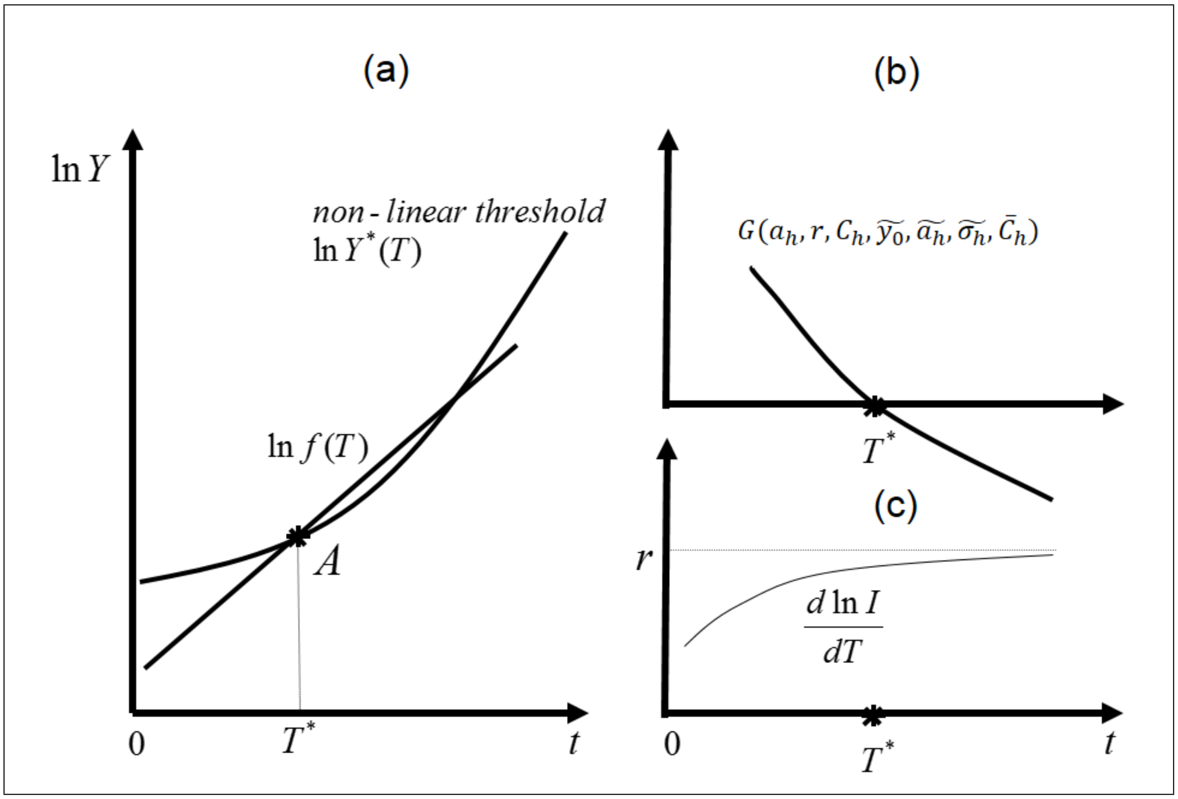

Figure B·1: Value and Distance Function

B.3.2 Existence of and Solution to T^*

Proof of Proposition 6. In general we look for the conditions described in Figure B·1. The threshold starts above the entry-level wage curve. For positive T the threshold will have a unique intersection with the expected first-time realization of the entry-level wage curve from below at A. Hence, at the time of expected market entry denoted by T^* the two curves $Y^*(T)$ and $f(T)$ intersect, so that $G := Y^*(T) - f(T) = 0$. G has to have a negative slope $\frac{dG}{dT} < 0$.Further, at T^* the threshold

$Y^*(T = 0)$ must start above $f(T)$, and $G > 0$ during the pre-market entry period $(0 < t < T^*)$. Otherwise the market entry would have taken place. ■

Negative Slope of G

$$\frac{\partial G}{\partial T^*} = \frac{\beta_h}{\beta_h - 1}(r - a_h)C_h e^{rT^*} - \tilde{y}_0 e^{T^*(\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)} < 0.$$

Before market entry, $f(T)$ must grow faster than the threshold curve. Only with a negative slope, G can approach and eventually reach zero. $\frac{\partial G}{\partial T} < 0$ is fulfilled if condition $\bar{C}_h > \frac{C_h}{r}$ (condition 3.12) holds.

Existence of an Intersection Point

a) With condition (3.12) the function $\ln Y^*(T)$ is convex. The function $\ln f(T)$ is linear, so that there are at most two intersections. We are interested only in intersections at $T > 0$. An intersection for positive values of both functions exists if conditions (3.11) and (3.12) hold, and if $G = 0$ for positive values of T^*.

b) Further, in Figure B·1, the condition for an intersection and a negative slope have to hold simultaneously at T^*. We need to show that there is a T^*, where both $\frac{dG}{dT} < 0$ and $G = 0$ hold. That is, we can find a minimum level for $\tilde{Y}(0)$ in order to ensure an intersection and a negative slope. Finally, $\tilde{Y}(0)$ has to lie in the open interval $\left(\frac{\beta_h}{\beta_h - 1}(r - a_h)\bar{C}_h \ , \ \frac{\beta_h}{\beta_h - 1}(r - a_h)\bar{C}_h \left(\frac{\frac{C_h}{\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2} - \frac{C_h}{r}}{-\frac{C_h}{r} + \bar{C}_h} \right)^{\frac{(\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)}{r}} \right)$.

T^* as an Implicit Function

If

(i) condition (3.11) holds,

(ii) the derivative $\frac{\partial G}{\partial T}$ is negative for each vector $(a_h, r, T^*, C_h, \tilde{y}_0, \tilde{a}_h, \tilde{\sigma}_h, \bar{C}_h)$ and

(iii) the partial derivatives of G with respect to $a_h, T^*, C_h, \tilde{y}_0, \tilde{a}_h, \tilde{\sigma}_h, \bar{C}_h$ and r are continuous (vide infra), we can apply the implicit function theorem. Hence, for a marginal environment of any vector $(a_h, r, C_h, \tilde{y}_0, \tilde{a}_h, \tilde{\sigma}_h, \bar{C}_h)$, T^* is an implicit function with

$$T^* = T^*(a_h, r, C_h, \tilde{y}_0, \tilde{a}_h, \tilde{\sigma}_h, \bar{C}_h).$$

B.4 Partial Derivatives of T^*

To apply comparative statics for the implicit function $T^* = T^*(a_h, r, C_h, \tilde{y}_0, \tilde{a}_h, \tilde{\sigma}_h, \bar{C}_h)$, we need to consider

$$\frac{\partial G}{\partial T} = \frac{\beta_h}{\beta_h - 1}(r - a_h)C_h e^{rT} - (\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)\tilde{y}_0 e^{(\tilde{a}_h - \frac{1}{2}\tilde{\sigma}_h^2)T} \gtrless 0.$$

We are only interested in values of T^* described by point A in Figure B·1, conditions (3.11) and $\frac{\partial G}{\partial T^*} < 0$. Then at T^* we obtain

a)

$$\frac{dT^*}{d\tilde{\sigma}_h} = -\frac{\tilde{\sigma}_h T^* \tilde{y}_0 e^{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)T^*}}{\frac{\beta_h}{\beta_h - 1}(r - \alpha_h)C_h e^{rT^*} - (\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)\tilde{y}_0 e^{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)T^*}} > 0,$$

b)

$$\frac{dT^*}{d\alpha_h} = \frac{[e^{rT^*} - 1 + \frac{\bar{C}_h r}{C_h}]}{\left(e^{rT} - \frac{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)(\beta_h - 1)}{(r - \alpha_h)\beta_h}\frac{\tilde{y}_0}{C_h}e^{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)T}\right)}\frac{1}{(r - \alpha_h)r} < 0,$$

c)

$$\frac{dT^*}{d\tilde{y}_0} = \frac{1}{\frac{\beta_h}{\beta_h - 1}(r - \alpha_h)C_h e^{\left(r - \tilde{\alpha}_h + \frac{1}{2}\sigma^2\right)T^*} - (\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)\tilde{y}_0} < 0,$$

d)

$$\frac{dT^*}{dC_h} = \frac{-[e^{rT^*} - 1]}{\left(e^{rT} - \frac{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)(\beta_h - 1)}{(r - \alpha_h)\beta_h}\frac{\tilde{y}_0}{C_h}e^{(\tilde{\alpha}_h - \frac{1}{2}\tilde{\sigma}_h^2)T}\right) rC_h} > 0.$$

B.5 Sheepskin Effects and Large Events

The earnings streams before entering the market are described by a geometric Brownian motion $d\tilde{Y}_i = \tilde{\alpha}_{h_i}\tilde{Y}_i + \tilde{\sigma}_{h_i}\tilde{Y}_i d\tilde{W}_{h_i}$. Since we distinguish two different education levels $i = 1, 2$ we assume two processes starting at different levels.[58] For the first development of education levels that start at $t = 0$ we assume

$$\tilde{Y}_1(0) = 0.$$

The second education level can only be started after finishing the first stage and after obtaining the sheepskin effect S_1. Therefore, after transforming the decision problem to $t = 0$,[59] we obtain

$$\tilde{Y}_2(0) = \tilde{Y}_1(T_1) + S_1.$$

The earning streams Y_1, Y_2 after market entry follow an Ito-Lévy Jump Diffusion process $\frac{dY_i}{Y_i} = \alpha_{h_i}dt + \sigma_{h_i}dW_h + \int\limits_{U_{h_i}} z_{h_i}N_{h_i}(t, dz_{h_i})$. A more general formulation of this process can be found in Oksendal and Sulem (2007). They describe under which conditions a solution to these SDE exists and discuss some characteristics. For our purpose, we assume that the existence conditions are fulfilled. A further discussion of Lévy processes and their characteristics can be found in, e.g. Applebaum (2009) and in Cont and Tankov (2004).

---

[58] Since we would like to show that there is a general relationship between sheepskin effects and the decision to enter the labour market, we keep the mathematical formulation as simple as possible. However, the more realistical model would assume that after finishing the first education level the earning stream before market entry in the second stage evolves according to the jump process Y_1 described below.

[59] In reality the transition takes place at $t = T_1$.

B.5.1 Solution to the SDE for $Y_i(t)$

Similar to the simple geometric Brownian motion case, define $X_i(t) = \ln Y_i(t)$ and use Ito's Lemma for jump processes in order to find the solution to the SDE.[60] The solution is

$$Y_i(t) = Y_i(0) \exp \left[\begin{array}{c} \left(\alpha_{h_i} - \frac{1}{2}\sigma_{h_i}^2\right) t + \sigma_{h_i} W_{h_i}(t) + \int_0^t \int_{U_{h_i}} \left[\ln(1 + z_{h_i}) - z_{h_i}\right] \upsilon_{h_i}(dz_{h_i}) ds \\ + \int_0^t \int_{U_{h_i}} \ln(1 + z_{h_i}) N_{h_i}(ds, dz_{h_i}) \end{array} \right].$$

B.5.2 ENPV and Option Value of $Y_i(t)$

Under the assumption that the Lévy processes included in the SDE, namely the Wiener and the compound Poisson process, are independent, the expected value of Y_i can be decomposed into

$$\begin{aligned} EY_i(t) = & \underbrace{Y_i(0) \cdot E e^{\left(\alpha_{h_i} - \frac{1}{2}\sigma_{h_i}^2\right)t + \sigma_{h_i} W_{h_i}(t)}}_{(1)} \cdot \underbrace{E\left[\exp\left(\int_0^t \int_{U_{h_i}} \ln(1 + z_{h_i}) N_{h_i}(ds, dz_{h_i})\right)\right]}_{(2)} \\ & \underbrace{\cdot E\left[\exp\left(\int_0^t \int_{U_{h_i}} \left[\ln(1 + z_{h_i}) - z_{h_i}\right] \upsilon_{h_i}(dz_{h_i}) ds\right)\right]}_{(3)}, \end{aligned}$$

so that we can compute the respective values for all three components individually.

- The expected value (1) is the same as for the geometric Brownian motion.
- In order to derive the expected value (2), we use Theorem 2.3.7 (i) in Applebaum (2009). $\int_0^t \int_{U_{h_i}} \ln(1 + z_{h_i}) N_{h_i}(ds, dz_{h_i})$ is compound Poisson distributed

[60] See also Oksendal and Sulem (2007).

with the characteristic function

$$E\left[\exp\left(iu_h\int_0^t\int_{U_{h_i}}\ln(1+z_{h_i})N_{h_i}(ds,dz_{h_i})\right)\right]=\exp\left(t\int_{U_{h_i}}\left(e^{ju_hz_{h_i}}-1\right)\upsilon_{if_h}(dz_{h_i})\right),$$

with $u_h\in\mathbb{C}$. It follows from $\upsilon_{if_h}=\upsilon_i\circ f_h^{-1}$, $f_h=\ln(1+z_{h_i})$ and $u_h=-j$ (imaginary unit)

$$E\left[\exp\left(\int_0^t\int_{U_{h_i}}\ln(1+z_{h_i})N_{h_i}(ds,dz_{h_i})\right)\right]=\exp\left(t\int_{f_h^{-1}(U_{h_i})}z_{h_i}\upsilon_{h_i}(dz_{h_i})\right).$$

This result only holds for $\int_{U_{h_i}}e^{u_hz_{h_i}}\upsilon_{h_i}(dz_{h_i})<\infty$.

- As the expected value (3) is given by

$$\begin{aligned}&E\left[\exp\left(\int_0^t\int_{U_{h_i}}\left[\ln(1+z_{h_i})-z_{h_i}\right]\upsilon_{h_i}(dz_{h_i})ds\right)\right]\\=\;&\exp\left(t\int_{U_{h_i}}\left[\ln(1+z_{h_i})-z_{h_i}\right]\upsilon_{h_i}(dz_{h_i})\right)\end{aligned}$$

  the resulting expected value for Y_i is

$$EY_i(t)=Y_i(0)\exp\left[t\left(\alpha_i+\int_{f^{-1}(U_{h_i})}z_{h_i}\upsilon_{h_i}(dz_{h_i})+\int_{U_{h_i}}\left[\ln(1+z_{h_i})-z_{h_i}\right]\upsilon_{h_i}(dz_{h_i})\right)\right].$$

$EY_i(t)$ is an increasing function for

$$\alpha_{h_i} + \int\limits_{f_h^{-1}(U_{h_i})} z_{h_i} \upsilon_{h_i}(dz_{h_i}) + \int\limits_{U_{h_i}} \left[\ln(1+z_{h_i}) - z_{h_i}\right] \upsilon_{h_i}(dz_{h_i}) > 0,$$

otherwise it decreases with t.

Expected Net Present Value

The Expected Net Present Values of the earnings streams for $i = 1, 2$ after assuming that $r > \alpha_{h_i} + \int\limits_{f_h^{-1}(U_{h_i})} z_{h_i} \upsilon_{h_i}(dz_{h_i}) + \int\limits_{U_{h_i}} \left[\ln(1+z_{h_i}) - z_{h_i}\right] \upsilon_{h_i}(dz_{h_i})$, are determined by

$$V_{h_i} = \frac{Y_{h_1}(T)}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1+z_{h_1}) - z_{h_1}\right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1}\right)} - \int\limits_0^T C_{h_1} e^{r(T-t)} dt - \bar{C}_{h_1}$$

and

$$V_{h_2} = \frac{Y_{h_2}(T)}{\left(r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) - \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2}\right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2}\right)} - \int\limits_{T_1}^T C_{h_2} e^{r(T-t)} dt - \bar{C}_{h_2}.$$

Option Value of a Later Market Entry

Similar to the simple geometric Brownian motion case, the Hamilton-Jacobi-Bellman equation holds for the option value F_{h_i} and $i = 1, 2$.

As a first step, we derive the option value for $i = 2$. We assume that the second education period starts at T_1 and ends at T_2. At T_2, the student obtains a premium for finishing this education level, the so-called sheepskin effect S_2. For any $t \in (T_1, T_2]$, the sheepskin effect S_2 has a discounted value[61] of $(1 - rT_2 + rt)S_2$. Therefore the time effect of the sheepskin effect on the option value is $\frac{\partial F_{h_2}}{\partial t} = rS_2$.

Hence, after applying Ito's Lemma and using this time effect, we obtain

$$rS_2 + \tilde{\alpha}_{h_2}\tilde{Y}_2 \frac{\partial F_{h_2}}{\partial \tilde{Y}_2} + \frac{1}{2}\tilde{\sigma}_{h_2}^2 \tilde{Y}_2^2 \frac{\partial^2 F_{h_2}}{\partial \tilde{Y}_2 \partial \tilde{Y}_2} - rF_{h_2} = 0.$$

This is a second-order inhomogeneous differential equation with a free boundary and has the solution

$$F_{h_2} = K_{h_2} \tilde{Y}_2^{\beta_{h_2}} + S_2.$$

Hence, after inserting this solution, we obtain $\beta_{h_2} = \frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}_{h_2}^2} + \sqrt{\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}_{h_2}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_2}^2}}$, which is similar to (3.9). The derivative with respect to $\tilde{\sigma}_2$ is

$$\frac{d\beta_{h_2}}{d\tilde{\sigma}_{h_2}^2} = \frac{2\tilde{\alpha}_{h_2}\left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}_{h_2}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_2}^2}\right]^{-\frac{1}{2}}}{\tilde{\sigma}_{h_2}^3} \left[\begin{array}{c} \left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}_{h_2}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_2}^2}\right]^{\frac{1}{2}} \\ + \frac{1}{2} - \frac{\tilde{\alpha}_{h_2}}{\tilde{\sigma}_{h_2}^2} - \frac{r}{\tilde{\alpha}_{h_2}} \end{array}\right] < 0.$$

In the next step, we derive the option value for $i = 1$. The first education period starts at 0 and ends at T_1. Staying in the education system until T_1 offers the sheepskin effect S_1 which for all $t \in [0, T_1]$ has the discounted value of $(1 - rT_1 + rt)S_1$. Furthermore, it enables the student to start and finish the second education level as well. Therefore, the option value of the first education period F_1 has to include the option value of the second education level F_{h_2}. Hence, the time effect of the option value is determined by $\frac{\partial F_{h_1}}{\partial t} = r\left(S_1 + F_{h_2}\right)$. The respective second-order inhomoge-

[61] The discounted value of the sheepskin effect is obtained by using continuous discounting. The exponential function is approximated by its Taylor expansion of the first degree.

neous differential equation with a free boundary becomes

$$r\left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}}\right) + \tilde{\alpha}_{h_1} \tilde{Y}_1 \frac{F_{h_1}}{\partial \tilde{Y}_1} + \frac{1}{2} \tilde{\sigma}_{h_1}^2 \tilde{Y}_1^2 \frac{\partial^2 F_{h_1}}{\partial \tilde{Y}_1 \partial \tilde{Y}_1} - r F_{h_1} = 0.$$

This is the so-called inhomogeneous Euler differential equation which has the solution[62] $F_1 = K_{h_1} Y^{\beta_{h_1}} + r\left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}}\right)$ where the respective positive and negative roots obtained for the homogenous differential equation $\beta_{h_1}^{1,2} = \frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2} \pm \sqrt{(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2})^2 + \frac{2r}{\tilde{\sigma}_{h_1}^2}}$ with $\beta_{h_1}^1 > 1$ and $\beta_{h_1}^2 < 0$. Furthermore, $Y_2^{*\beta_{h_2}}$ is a constant obtained from the optimization problem for the second formal education level. The derivatives with respect to $\tilde{\sigma}_{h_1}^2$ and $\tilde{\sigma}_{h_1}^2$ are

$$\frac{\partial \beta_{h_1}^1}{\partial \tilde{\sigma}_{h_1}^2} = \frac{2\tilde{\alpha}_{h_1}\left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_1}^2}\right]^{-\frac{1}{2}}}{\tilde{\sigma}_{h_1}^3} \left[\left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_1}^2}\right]^{\frac{1}{2}} + \frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2} - \frac{r}{\tilde{\alpha}_{h_1}}\right] < 0,$$

$$\begin{aligned} \frac{\partial \beta_{h_1}^2}{\partial \tilde{\sigma}_{h_1}^2} &= \frac{2\tilde{\alpha}_{h_1}\left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_1}^2}\right]^{-\frac{1}{2}}}{\tilde{\sigma}_{h_1}^3} \left[\left[\left(\frac{1}{2} - \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2}\right)^2 + \frac{2r}{\tilde{\sigma}_{h_1}^2}\right]^{\frac{1}{2}} - \frac{1}{2} + \frac{\tilde{\alpha}_{h_1}}{\tilde{\sigma}_{h_1}^2} + \frac{r}{\tilde{\alpha}_{h_1}}\right] \\ &< 0 \quad \text{for } r > \tilde{\alpha}_{h_1}. \end{aligned}$$

B.5.3 Entry Thresholds Y_1^* and Y_2^*

Proof of Proposition 8. With the same method to obtain the threshold for the simple geometric Brownian motion case, we compute the threshold for the second

[62] In order to obtain the solution, transform the inhomogenous differential equation in an inhomogenous linear differential equation with constant coefficients. This can be done with $\check{Y} = \exp(t)$. Next, find a solution to this equation by means of the characteristic polynomial and variation of constants. Finally, transform the solution back to the original variable.

education level $i = 2$. Now we obtain the threshold Y_2^* for $i = 2$

$$Y_2^* = \left(\begin{array}{c} r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) \\ - \int\limits_{U_{h_2}} \left[\ln(1 + z_{h_2}) - z_{h_2} \right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2} \end{array} \right) \frac{\beta_{h_2}}{\beta_{h_2} - 1} \left(I_{h_2} + S_2 \right).$$

■

Proof of Proposition 9. For the first education level $i = 1$, we obtain

$$\begin{aligned} & \frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{Y_1^*}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1 + z_{h_1}) - z_{h_1} \right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)} \\ = \; & r \left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}} \right) - I_{h_1}. \end{aligned}$$

Since this is a non-linear equation, the solution can be found by using the implicit function theorem after defining

$$\begin{aligned} H \; : \; = & \frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{Y_1^*}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1 + z_{h_1}) - z_{h_1} \right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)} \\ & - r \left(S_1 + S_2 + K_{h_2} Y_2^{*\beta_{h_2}} \right) - I_{h_1}. \end{aligned}$$

■

Derivative with respect to Y_1^*

In a first step, we show that the function H has a non-zero slope according to Y_1^*

$$\frac{\partial H(Y_1^*)}{\partial Y_1^*} = \frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{1}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} [\ln(1+z_{h_1}) - z_{h_1}]\, \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)}$$
$$> 0.$$

The derivative is positive due to $\beta_{h_1}^1 > 1$ and

$$r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} [\ln(1+z_{h_1}) - z_{h_1}]\, \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} > 0,$$

that is, it is not equal to zero.

Regular Value

Now we need the notion "regular value". A differentiable function f has the regular value y if for all $x \in f^{-1}(y)$ the derivative $Df(x)$ has a full rank. As the derivative of H with respect to Y_1^* is non-zero, 0 is a regular value of $H : \mathbb{R}^{13} \to \mathbb{R}$ and the set of points $H^{-1}(0)$ is a manifold of dimension 12 (see Milnor, 1997, p. 11).

Implicit Function

As $H^{-1}(0)$ is a manifold and as for each vector

$$(Y_1^*, r, z_{h_1}, \alpha_{h_1}, \tilde{\alpha}_{h_1}, \tilde{\alpha}_{h_2}, \tilde{\sigma}_{h_1}, \tilde{\sigma}_{h_2}, S_1, S_2, C_{h_1}, \bar{C}_{h_1}, \lambda_{h_1})$$

the derivative

$$\frac{\partial H(Y_1^*)}{\partial Y_1^*}(Y_{10}^*, r_0, z_{h_{10}}, \alpha_{h_{10}}, \tilde{\alpha}_{h_{10}}, \tilde{\alpha}_{h_{20}}, \tilde{\sigma}_{h_{10}}, \tilde{\sigma}_{h_{20}}, S_{10}, S_{20}, C_{h_{10}}, \bar{C}_{h_{10}}, \lambda_{h_{10}})$$

is non-negative and the partial derivatives according

$$Y_1^*, r, z_{h_1}, \alpha_{h_1}, \tilde{\alpha}_{h_1}, \tilde{\alpha}_{h_2}, \tilde{\sigma}_{h_1}, \tilde{\sigma}_{h_2}, S_1, S_2, C_{h_1}, \bar{C}_{h_1}, \lambda_{h_1}$$

are continuous, we can apply the implicit function theorem. Hence, for a marginal environment of any vector

$$(Y_{10}^*, r_0, z_{h_{10}}, \alpha_{h_{10}}, \tilde{\alpha}_{h_{10}}, \tilde{\alpha}_{h_{20}}, \tilde{\sigma}_{h_{10}}, \tilde{\sigma}_{h_{20}}, S_{10}, S_{20}, C_{h_{10}}, \bar{C}_{h_{10}}, \lambda_{h_{10}}),$$

Y_1^* is an implicit function of $(r, z_{h_1}, \alpha_{h_1}, \tilde{\alpha}_{h_1}, \tilde{\alpha}_{h_2}, \tilde{\sigma}_{h_1}, \tilde{\sigma}_{h_2}, S_1, S_2, C_{h_1}, \bar{C}_{h_1})$.

B.5.4 Derivatives of Y_1^* and Y_2^*

Proof of Proposition 11.

$$\begin{aligned}
\frac{\partial Y_1^*}{\partial S_1} &= -\frac{-r}{\underbrace{\frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{1}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1+z_{h_1}) - z_{h_1} \right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)}}_{(+)}} \\
&> 0,
\end{aligned}$$

$$\frac{\partial Y_1^*}{\partial S_2} = -\frac{-r \left[1 + \begin{pmatrix} r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) \\ - \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2} \right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2} \end{pmatrix}^{\frac{\beta_{h_1}}{\beta_{h_1} - 1}} \right]}{\frac{\beta_{h_1}^1 - 1}{\beta_{h_1}^1} \frac{1}{\left(r - \int\limits_{f_h^{-1}(U_{h_1})} z_{h_1} \upsilon_{h_1}(dz_{h_1}) - \int\limits_{U_{h_1}} \left[\ln(1+z_{h_1}) - z_{h_1} \right] \upsilon_{h_1}(dz_{h_1}) - \alpha_{h_1} \right)}} > 0.$$

■

Proof of Proposition 12.

$$\frac{\partial Y_2^*}{\partial \tilde{\sigma}_2} = -\left(\begin{array}{c} r - \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} \upsilon_{h_2}(dz_{h_2}) \\ - \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2} \right] \upsilon_{h_2}(dz_{h_2}) - \alpha_{h_2} \end{array} \right) \frac{\frac{\partial \beta_{h_1}}{\partial \tilde{\sigma}_{h_2}}}{\left(\beta_{h_1} - 1\right)^2} (I_2 + S_2) > 0,$$

$$\frac{\partial Y_2^*}{\partial \lambda_{h_2}} = -\left(\begin{array}{c} \int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} g_{h_2}(dz_{h_2}) \\ + \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2} \right] g_{h_2}(dz_{h_2}) \end{array} \right) \frac{\beta_{h_1}}{\beta_{h_1} - 1} (I_2 + S_2).$$

Note that the derivative has been computed after replacing υ_{h_2} through $\lambda_{h_2} g_{h_2}$. λ_{h_2} is the jump intensity of N_{h_2} and g_{h_2} denotes the distribution of jump sizes. The sign of the derivative depends on whether opportunities overweigh threats so that $\int\limits_{f_h^{-1}(U_{h_2})} z_{h_2} g_{h_2}(dz_{h_2}) + \int\limits_{U_{h_2}} \left[\ln(1+z_{h_2}) - z_{h_2} \right] g_{h_2}(dz_{h_2}) > 0$. In this case the sign is negative and otherwise positive. ■

Appendix C

Uncertainty and Conflict Decision

C.1 Solution to and Expected Value of the SDE

In this section, we determine the solution of the SDE for B and derive the expected value. In order to obtain the respective results for $\tilde{B}$, replace B by $\tilde{B}$.

C.1.1 Solution to the SDE for B

The Ito-Lévy Jump Diffusion process described by the SDE

$$dB = \alpha_s B dt + \sigma_s B dW_s + B \int_{U_s} z_s N_s(t, dz_s) \quad \text{for} \quad T < t,$$

has the solution

$$B(t) = B(0) \exp \left[\begin{array}{c} \left(\alpha_s - \frac{1}{2}\sigma_s^2\right) t + \sigma_s W_s(t) + \int_0^t \int_{U_s} \left[\ln(1 + z_s) - z_s\right] \upsilon_s(dz_s) ds \\ + \int_0^t \int_{U_s} \ln(1 + z_s) N_s(dt, dz_s) \end{array} \right].$$

This can be derived by defining $X(t) := \ln B(t)$ and using Ito's Lemma for jump processes to obtain the solution to the SDE. A similar procedure can be found in Oksendal and Sulem (2007).

C.1.2 Expected Value of B

Let B be an Ito-Lévy Jump Diffusion process. The expected value of B is

$$EB(t) = B(0)\exp\left[t\left(\alpha_s + \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) + \int\limits_{U_s} \left[\ln(1+z_s) - z_s\right]\upsilon_s(dz_s)\right)\right].$$

To show this, assume that the Wiener and the compound Poisson process are independent. Then the expected value of B can be decomposed into

$$\begin{aligned} EB(t) &= \underbrace{B(0)\cdot Ee^{\left(\alpha_s - \frac{1}{2}\sigma_s^2\right)t + \sigma_s W_s(t)}}_{(1)} \cdot \underbrace{E\left[\exp\left(\int\limits_0^t\int\limits_{U_s} \ln(1+z_s)N_s(ds,dz_s)\right)\right]}_{(2)} \\ &\quad \underbrace{\cdot E\left[\exp\left(\int\limits_0^t\int\limits_{U_s}\left[\ln(1+z_s) - z_s\right]\upsilon_s(dz_s)ds\right)\right]}_{(3)}. \end{aligned}$$

In this case compute the respective values for all three components.

The expectation value (1) is the same as for the geometric Brownian motion, which can be found in Dixit and Pindyck (1994). It is

$$B(0)e^{\alpha_s t}.$$

In order to compute the expected value (2), we use Theorem 2.3.7 (i) in Applebaum (2009). As $\int\limits_0^t\int\limits_{U_s} \ln(1+z_s)N_s(ds,dz_s)$ is compound Poisson distributed with the characteristic function

$$E\left[\exp\left(id\int\limits_0^t\int\limits_{U_s} \ln(1+z_s)N_s(ds,dz_s)\right)\right] = \exp\left(t\int\limits_{U_s}\left(e^{idz_s} - 1\right)\upsilon_{sf}(dz_s)\right),$$

with $d \in \mathbb{C}$, $\upsilon_{sf} = \upsilon_s \circ f_s^{-1}$, $f_s = \ln(1+z_s)$ and $d = -i$.

$$E\left[\exp\left(\int_0^t \int_{U_s} \ln(1+z_s) N_s(ds, dz_s)\right)\right] = \exp\left(t \int_{f^{-1}(U_s)} z_s \upsilon_s(dz_s)\right).$$

This result only holds for $\int_{U_s} e^{dz_s} \upsilon_s(dz_s) < \infty$.

As the expected value (3) is given by

$$E\left[\exp\left(\int_0^t \int_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s) ds\right)\right] = \exp\left(t \int_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s)\right).$$

Accordingly, the resulting expected value for B is

$$EB(t) = B(0) \exp\left[t\left(\alpha_s + \int_{f^{-1}(U_s)} z_s \upsilon_s(dz_s) + \int_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s)\right)\right].$$

$EB(t)$ is an increasing function for $\alpha_s + \int_{f^{-1}(U_s)} z_s \upsilon_s(dz_s) + \int_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s) > 0$, otherwise it decreases with t.

C.2 Value of Conflict and Option Value of Peacekeeping

In order to determine the optimal investment in rebellion, the value of conflict and the option value of waiting to attack are optimized for each period.

Proof of Proposition 15. For the value of conflict, all benefits per period are summarized through an integral. Similar to Dixit and Pindyck (1994) we obtain

$$V_s^{gross} = E \int_T^\infty B e^{-r(t-T)} dt.$$

Under the assumption $r > \alpha_s + \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) + \int\limits_{U_s} [\ln(1+z_s) - z_s]\, \upsilon_s(dz_s)$ we obtain the result. ■

Corollary 20 *Let $\tilde{B}$ be an Ito-Lévy Jump Diffusion process. Then the Hamilton-Jacobi-Bellman equation defined in Dixit and Pindyck (1994)*

$$rF_s = \frac{1}{dt} E(dF_s)$$

has the solution $F_s = K_s \tilde{B}^\beta$ where β is defined implicitly.

Proof. From Ito's Lemma we know

$$\begin{aligned} dF_s &= \left(\frac{\partial F_s}{\partial t} + \tilde{\alpha}_s \tilde{B} \frac{\partial F_s}{\partial \tilde{B}} + \frac{1}{2} \tilde{\sigma}_s^2 \tilde{B}^2 \frac{\partial^2 F_s}{\partial \tilde{B} \partial \tilde{B}}\right) dt + \tilde{\sigma}_s \tilde{B} \frac{\partial F_s}{\partial \tilde{B}} d\tilde{W}_s \\ &+ \int\limits_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) - \frac{\partial F_s}{\partial \tilde{B}} u_s \tilde{B}(t^-) \right] \tilde{\upsilon}_s(du_s) dt \\ &+ \int\limits_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) \right] \tilde{N}_s(dt, du_s), \end{aligned}$$

with $\tilde{B}(t^-)$ denoting the left limit of $\tilde{B}$ in t. In order to determine $E(dF_s)$, we use Theorem 2.3.7 (ii) in Applebaum (2009). For the expectation value of

$$\int\limits_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) \right] \tilde{N}_s(dt, du_s)$$

we obtain

$$\begin{aligned} & E \int\limits_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) \right] \tilde{N}_s(dt, du_s) \\ = \;& t \int\limits_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) \right] \tilde{\upsilon}_s(du_s). \end{aligned}$$

and with $E(d\tilde{W}_s) = 0$, this leads us to

$$\Rightarrow \quad E(dF_s) = (\frac{\partial F_s}{\partial t} + \tilde{\alpha}_s \tilde{B} \frac{\partial F_s}{\partial \tilde{B}} + \frac{1}{2}\tilde{\sigma}_s^2 \tilde{B}^2 \frac{\partial^2 F_s}{\partial \tilde{B} \partial \tilde{B}})dt$$
$$+2\int_{\tilde{U}_s} \left[F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) - \frac{1}{2}\frac{\partial F_s}{\partial \tilde{B}} u_s \tilde{B}(t^-) \right] \tilde{\upsilon}_s(du_s)dt.$$

From the Bellman and the last equation, we obtain the following differential equation

$$\tilde{\alpha}_s \tilde{B} \frac{\partial F_s}{\partial \tilde{B}} + \frac{1}{2}\tilde{\sigma}_s^2 \tilde{B}^2 \frac{\partial^2 F_s}{\partial \tilde{B} \partial \tilde{B}} + 2\int_{\tilde{U}_s} \left[\begin{array}{c} F_s(\tilde{B}(t^-) + u_s \tilde{B}(t^-)) - F_s(\tilde{B}(t^-)) \\ -\frac{1}{2}\frac{\partial F_s}{\partial \tilde{B}} u_s \tilde{B}(t^-) \end{array} \right] \tilde{\upsilon}_s(du_s) - rF_s = 0.$$

This is a second-order homogenous ordinary differential equation with a free boundary. A general solution to this differential equation is of the form

$$F_s = K_s \tilde{B}^{\beta_s}.$$

Hence,

$$\tilde{\alpha}_s \beta_s + \frac{1}{2}\tilde{\sigma}_s^2 \beta_s(\beta_s - 1) + 2\int_{\tilde{U}_s} \left[(1+u_s)^{\beta_s} - (1 + \frac{1}{2}\beta_s u_s) \right] \tilde{\upsilon}_s(du_s) - r = 0.$$

If we define

$$g(\beta_s) := \tilde{\alpha}_s \beta_s + \frac{1}{2}\tilde{\sigma}_s^2 \beta_s(\beta_s - 1) + 2\int_{\tilde{U}_s} \left[(1+u_s)^{\beta_s} - (1 + \frac{1}{2}\beta_s u_s) \right] \tilde{\upsilon}_s(du_s) - r,$$

then it follows

$$\begin{aligned} g(1) &= \tilde{\alpha}_s + \int\limits_{\tilde{U}_s} u_s \tilde{\upsilon}_s(du_s) - r, \\ \lim_{\beta \to \infty} g(\beta) &= \infty. \end{aligned}$$

Accordingly, we can assume that $r > \tilde{\alpha}_s + \int\limits_{\tilde{U}_s} u_s \tilde{\upsilon}_s(du_s)$ leading to $g(1) < 0$. With the intermediate value theorem we find $\beta_s \in (1, \frac{r}{\tilde{\alpha}_s + \int\limits_{\tilde{U}_s} u_s \tilde{\upsilon}_s(du_s)})$, such that $g(\beta_s) = 0$. It follows immediately that β_s is a function of r, $\tilde{\alpha}_s$ and $\int\limits_{\tilde{U}_s} u_s \tilde{\upsilon}_s(du_s)$, which was determined as the implicit function of $g(\beta_s) = 0$, and $\beta_s > 1$. ■

Corollary 21 *The derivatives of β_s with respect to $\tilde{\sigma}_s, \tilde{\lambda}_s$ and u_s are*

$$\frac{\partial \beta_s}{\partial \tilde{\sigma}_s} = -\frac{\tilde{\sigma}_s \beta_s (\beta_s - 1)}{\tilde{\alpha}_s - \frac{1}{2}\tilde{\sigma}_s^2 + 2\tilde{\lambda}_s \int\limits_{\tilde{U}_s} \left[\ln(1+u_s)(1+u_s)^{\beta_s} - \frac{1}{2}u_s\right] \tilde{\upsilon}_s(du_s)} < 0,$$

$$\frac{\partial \beta_s}{\partial \tilde{\lambda}_s} = \frac{2\int\limits_{\tilde{U}_s} \left[-(1+u_s)^{\beta_s} + (1 + \frac{1}{2}\beta_s u_s)\right] h_s(du_s)}{\tilde{\alpha}_s - \frac{1}{2}\tilde{\sigma}_s^2 + 2\tilde{\lambda}_s \int\limits_{\tilde{U}_s} \left[\ln(1+u_s)(1+u_s)^{\beta_s} - \frac{1}{2}u_s\right] h_s(du_s)}$$

and

$$\frac{\partial \beta_s}{\partial u_s} = -\frac{2\beta_s \int\limits_{\tilde{U}_s} \left[(1+u_s)^{\beta_s - 1} - \frac{1}{2}\right] \tilde{\upsilon}_s(du_s)}{\tilde{\alpha}_s - \frac{1}{2}\tilde{\sigma}_s^2 + 2\tilde{\lambda}_s \int\limits_{\tilde{U}_s} \left[\ln(1+u_s)(1+u_s)^{\beta_s} - \frac{1}{2}u_s\right] \tilde{\upsilon}_s(du_s)},$$

with h_s denoting the distribution of jump sizes.

Proof. Apply the rules

$$\frac{\partial \beta_s}{\partial \tilde{\sigma}_s} = -\frac{\frac{\partial g}{\partial \tilde{\sigma}_s}}{\frac{\partial g}{\partial \beta_s}}, \frac{\partial \beta_{s_1}}{\partial \tilde{\lambda}_s} = -\frac{\frac{\partial g}{\partial \lambda}}{\frac{\partial g}{\partial \beta_s}} \text{ and } \frac{\partial \beta_s}{\partial u_s} = -\frac{\frac{\partial g}{\partial u_s}}{\frac{\partial g}{\partial \beta_s}}$$

to obtain the derivatives. Their sign is obtained by discussing the jump integral. The numerator of $\frac{\partial \beta_s}{\partial \tilde{\lambda}_s}$ contains a measure integral

$$\int_U \left[-(1+u_s)^{\beta_s} + (1 + \frac{1}{2}\beta_s u_s) \right] h(du_s),$$

with a measurable function

$$f(u_s) := -(1+u_s)^{\beta_s} + (1 + \frac{1}{2}\beta_s u_s)$$

and a measure h. If the negative jumps outweigh the positive jumps, the sign of the integral will be negative and otherwise positive. In our case, where we have repressive actions that lead to a positive jump in the benefits, the integral is positive as well. In the denominator, we have

$$\tilde{\alpha}_s - \frac{1}{2}\tilde{\sigma}_s^2 + 2\tilde{\lambda}_s \int_{\tilde{U}_s} \ln(1+u_s)(1+u_s)h_s(du_s) - \tilde{\lambda}_s \int_{\tilde{U}_s} u_s h_s(du_s),$$

consisting of

$$\tilde{\alpha}_s - \frac{1}{2}\tilde{\sigma}_s^2 > 0$$

and a jump component

$$2\tilde{\lambda}_s \int_{\tilde{U}_s} \ln(1+u_s)(1+u_s)h_s(du_s) - \tilde{\lambda}_s \int_{\tilde{U}_s} u_s h_s(du_s),$$

where the logarithm is only defined for $u_s > -1$. Again, in our case, positive jumps lead to a positive integral. Assuming the denominator and numerator are positive, the derivative with respect to $\tilde{\lambda}_s$ becomes negative. With the same argumentation the derivative with respect to u_s becomes positive. ■

C.3 Investment Threshold and Expected Time of Conflict

Proof of Proposition 16. Apply the boundary conditions

$$\begin{aligned} F_s(0) &= 0, \\ F_s(B^*) &= V_s^{gross}(B^*) - I_s, \\ \frac{dF_s(B^*)}{dB} &= \frac{d(V_s^{gross}(B^*) - I_s)}{dB} \end{aligned}$$

and solve the equation system for B^*. ■

Proof of Proposition 17. For the jump diffusion case the first passage problem can be solved analytically if we assume an explicit distribution of the jump sizes. According to Kou and Wang (2003b), the moment-generating function for $\tilde{B}(t)$ with $\theta \in (0, \eta_s)$ is

$$\phi(\theta, t) := E(e^{\theta \tilde{B}(t)}) = \exp(G(\theta)t),$$

where the function G is defined as

$$G(x) := x\tilde{\alpha}_s + \frac{1}{2}x^2\tilde{\sigma}_s^2 + \tilde{\lambda}_s \left(\frac{\eta_s}{\eta_s - x} - 1 \right).$$

For jump diffusion processes, the study of first passage times has to consider the exact hit of a constant boundary as well as an overshoot. Accordingly, two cases have to be distinguished. The Laplace transform of the first hitting time, when $\tilde{B}(t)$ hits the boundary B^* exactly[63]is

$$E(e^{-\varepsilon \tilde{T}_{s_i}} 1_{\{\tilde{B}(\tilde{T}_{s_i})=B^*\}}) = \frac{\eta_s - \beta_{s_{1,\varepsilon}}}{\beta_{s_{2,\varepsilon}} - \beta_{s_{1,\varepsilon}}} e^{-B^*\beta_{1,\varepsilon}} + \frac{\beta_{s_{2,\varepsilon}} - \eta_s}{\beta_{s_{2,\varepsilon}} - \beta_{s_{1,\varepsilon}}} e^{-B^*\beta_{s_{2,\varepsilon}}},$$

with $\beta_{s_{1,\varepsilon}}$ and $\beta_{s_{2,\varepsilon}}$ being the only positive roots of $G(\beta) - \varepsilon$ and $0 < \beta_{1,\varepsilon} < \eta_s <$

[63] See Kou and Wang (2003a), Theorem 3.1.

$\beta_{2,\varepsilon} < \infty$. For every overshoot $\tilde{B}(\tilde{T}_s) - B^*$, the Laplace transform is

$$\begin{aligned} E(e^{-\varepsilon \tilde{T}_{s_i}} 1_{\{\tilde{B}(\tilde{T}_i)-B^*>y\}}) &= e^{-\eta_s y} \frac{\left(\eta_s - \mu_{s_{1,\varepsilon}}\right)\left(\mu_{s_{2,\varepsilon}} - \eta_s\right)}{\eta_s \left(\mu_{s_{2,\varepsilon}} - \mu_{s_{1,\varepsilon}}\right)} \left(e^{-B^* \mu_{s_{1,\varepsilon}}} - e^{-B^* \mu_{s_{2,\varepsilon}}}\right) \\ \text{for all } y &\geq 0. \end{aligned}$$

The expectation of the first passage time is finite, i.e., $E(T_s^*) < \infty$, if and only if the overall drift of the jump diffusion process is positive. Hence,

$$E(T^*) < \infty \Leftrightarrow \bar{u}_s = \tilde{\alpha}_s + \tilde{\lambda}_s \frac{1}{\eta_s} > 0.$$

Now for $\bar{u}_s > 0$ we determine the first passage time as

$$E(T^*) = \frac{1}{\bar{u}_s} \left[B^* + \frac{\mu_2^* - \eta_s}{\eta_s \mu_{s_2}^*} (1 - e^{-B \mu_{s_2}^*}) \right],$$

where $\mu_{s_2}^*$ is defined as the unique root of $G(\mu_{s_2}^*) = 0$ with $0 < \eta_s < \mu_{s_2}^* < \infty$. ∎

C.4 Determinants of the Expected Time of Conflict

Proof of Proposition 18. In order to determine the derivative of $E(T^*)$, we first have to find out the sign of $\frac{\partial B^*}{\partial \tilde{\sigma}_s}$ and $\frac{\partial \mu_2^*}{\partial \tilde{\sigma}_s}$.

$$\begin{aligned} \frac{\partial B^*}{\partial \tilde{\sigma}_s} &= \frac{\frac{\partial \beta_s}{\partial \tilde{\sigma}_s}(\beta_s - 1) - \frac{\partial \beta_s}{\partial \tilde{\sigma}_s}\beta}{(\beta_s - 1)^2} \begin{pmatrix} r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) \\ -\int\limits_{U_s} [\ln(1+z_s) - z_s] \upsilon_s(dz_s) - a_s \end{pmatrix} I_s \\ &= -\frac{\frac{\partial \beta_s}{\partial \tilde{\sigma}_s}}{(\beta_s - 1)^2} \begin{pmatrix} r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) \\ -\int\limits_{U_s} [\ln(1+z_s) - z_s] \upsilon_s(dz_s) - a_s \end{pmatrix} I_s > 0. \end{aligned}$$

Now, define $G := x\tilde{\alpha}_s + \frac{1}{2}x^2\tilde{\sigma}_s^2 + \lambda_s\left(\frac{\eta_s}{\eta_s - x} - 1\right)$ and determine

$$\begin{aligned}\frac{\partial\mu_2^*}{\partial\tilde{\sigma}_s} &= \frac{\partial x}{\partial\tilde{\sigma}_s} = -\frac{\frac{\partial G}{\partial\tilde{\sigma}_s}}{\frac{\partial G}{\partial x}} \\ &= -\frac{x^2\tilde{\sigma}_s}{\tilde{\alpha}_s + x\tilde{\sigma}_s^2 + \tilde{\lambda}_s\frac{\eta_s}{(\eta_s - x)^2}} < 0.\end{aligned}$$

With these results we can now determine

$$\begin{aligned}\frac{\partial E(T^*)}{\partial\tilde{\sigma}_s} &= \frac{1}{\bar{u}_s}\frac{\partial B^*}{\partial\tilde{\sigma}_s} + \frac{1}{\bar{u}_s}\left(\frac{\partial B^*}{\partial\tilde{\sigma}_s}\mu_{s_2}^* + B^*\frac{\partial\mu_{s_2}^*}{\partial\tilde{\sigma}_s}\right)\frac{\mu_{s_2}^* - \eta_s}{\eta_s\mu_{s_2}^*}e^{-B^*\mu_2^*} \\ &\quad + \frac{\frac{\partial\mu_{s_2}^*}{\partial\sigma}\eta_s\mu_{s_2}^* - \frac{\partial\mu_{s_2}^*}{\partial\sigma}\eta_s\mu_{s_2}^*}{\left(\eta_s\mu_{s_2}^*\right)^2}(1 - e^{-B^*\mu_{s_2}^*}) \\ &= \frac{1}{\bar{u}_s}\underbrace{\frac{\partial B^*}{\partial\tilde{\sigma}_s}}_{(+)} + \frac{1}{\bar{u}_s}\left(\underbrace{\frac{\partial B^*}{\partial\tilde{\sigma}_s}}_{(+)}\mu_{s_2}^* + B^*\underbrace{\frac{\partial\mu_{s_2}^*}{\partial\tilde{\sigma}_s}}_{(-)}\right)\frac{\mu_{s_2}^* - \eta_s}{\eta_s\mu_{s_2}^*}e^{-B^*\mu_{s_2}^*} > 0,\end{aligned}$$

with $B^*\underbrace{\frac{\partial\mu_{s_2}^*}{\partial\tilde{\sigma}_s}}_{(-)}$ sufficiently small. ■

Proof of Proposition 19.

$$\begin{aligned}\frac{\partial E(T^*)}{\partial\tilde{\lambda}_s} &= -\underbrace{\frac{\frac{1}{\eta_s}}{\left[\tilde{\alpha}_s + \tilde{\lambda}_s\frac{1}{\eta_s}\right]^2}}_{(1)}\underbrace{\left[B^* + \frac{\mu_{s_2}^* - \eta_s}{\eta_s\mu_{s_2}^*}(1 - e^{-B^*\mu_{s_2}^*})\right]}_{(2)} \\ &\quad + \underbrace{\frac{1}{\tilde{\alpha}_s + \tilde{\lambda}_s\frac{1}{\eta_s}}}_{(3)}\underbrace{\left[\begin{array}{c}-\frac{\frac{\partial\beta_{s_1}}{\partial\tilde{\lambda}_s}}{\left(\beta_{s_1} - 1\right)^2}(r - \int\limits_{f_s^{-1}(U_s)} z_s\upsilon_s(dz_s) \\ -\int\limits_{U_s}\left[\ln(1 + z_s) - z_s\right]\upsilon_s(dz_s) - a_s)I_s\end{array}\right]}_{(4)}\underbrace{(1 - \frac{\mu_{s_2}^* - \eta_s}{\eta_s}e^{-B^*\mu_{s_2}^*})}_{(5)}.\end{aligned}$$

For the first term (1) and term (3), we obtain

$$\frac{\frac{1}{\eta_s}}{\left[\tilde{\alpha}_s + \tilde{\lambda}_s \frac{1}{\eta_s}\right]^2} > 0$$

and

$$\frac{1}{\tilde{\alpha}_s + \tilde{\lambda}_s \frac{1}{\eta_s}} > 0.$$

For the second term (2), it holds that

$$\underbrace{B^*}_{>0} + \underbrace{\frac{\mu_{s_2}^* - \eta_s}{\eta_s \mu_{s_2}^*}}_{>0} \underbrace{(1 - e^{-B^* \mu_{s_2}^*})}_{\geq 0} > 0.$$

The sign of the fourth term (4) depends on whether $\frac{\partial \beta_{s_1}}{\partial \tilde{\lambda}_s}$ is positive or negative. Assuming $\frac{\partial \beta_{s_1}}{\partial \tilde{\lambda}_s} < 0$, then term (4) becomes

$$-\frac{\frac{\partial \beta_{s_1}}{\partial \tilde{\lambda}_s}}{\left(\beta_{s_1} - 1\right)^2} \underbrace{(r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) - \int\limits_{U_s} \left[\ln(1 + z_s) - z_s\right] \upsilon_s(dz_s) - a_s)}_{>0} \underbrace{I_s}_{>0} > 0.$$

The last term (5)

$$1 - \frac{\mu_2^* - \eta_s}{\eta_s} e^{-B^* \mu_{s_2}^*}$$

is negative if

$$1 < \frac{\mu_{s_2}^* - \eta_s}{\eta_s} e^{-B^* \mu_{s_2}^*}.$$

Summarizing all conditions leads to $\frac{\partial E(T^*)}{\partial \tilde{\lambda}_s} < 0$.

Proof.

$$\frac{\partial E(T^*)}{\partial u_s} = \underbrace{\frac{1}{\tilde{\alpha}_s + \tilde{\lambda}_s \frac{1}{\eta_s}}}_{(1)} \underbrace{\left[\begin{array}{c} -\frac{\frac{\partial \beta_{s_1}}{\partial u_s}}{\left(\beta_{s_1}-1\right)^2}(r - \int\limits_{f_s^{-1}(U_s)} z_s \upsilon_s(dz_s) \\ -\int\limits_{U_s} \left[\ln(1+z_s) - z_s\right] \upsilon_s(dz_s) - \alpha_s) I_s \end{array} \right]}_{(2)}$$

$$\underbrace{\cdot(1 - \frac{\mu^*_{s_2} - \eta_s}{\eta_s} e^{-B^* \mu^*_{s_2}})}_{(3)}.$$

Similar to the last proof, the term (1) is positive. Accordingly, the last component (3) is negative for $1 < \frac{\mu^*_{s_2} - \eta_s}{\eta_s} e^{-B^* \mu^*_{s_2}}$. The sign of (2) depends on whether $\frac{\partial \beta_{s_1}}{\partial u_s} \gtrless 0$. Assuming that $\frac{\partial \beta_{s_1}}{\partial u_s} < 0$ it follows that $\frac{\partial E(\tilde{T})}{\partial u_s} < 0$. ■