35

FIVE VALUED QUASI REAL BOOLEAN FUNCTIONS

Franz J. Rammig

Abstract

1. Short abstract

This paper describes a generalization of Quasi Real Boolean Functions. Boolean functions are expanded to functions of signals. Through this procedure it is possible to describe time-dependent phenomena of ohysically realized Boolean functions. These phenomena include dynamic variable delay, which is dependent on different values, value dependent inertial delay (high frequency absorption), transition sensitivity and transition smoothing. By practical reasons signals are characterized by a mapping of a time-set into a five-valued set FV. Five Valued Quasi Real Boolean Functions are well suited as foundation of highly sophisticated simulation systems.

2. Five valued signals

2.1 Let be (T,+,d) a group with metric. $S_T := \{o,p,l,n,u\}^T$ is called five valued signalset. If $T \hat{=} \mathbb{Z}$ we speak about discrete signals, if $T \hat{=} \mathbb{R}$ we speak of continuous signals. Let be $a = \langle a_t \rangle_{t \in T}$ denote a signal.

2.2 The semantics of the five signals is given informally as follows:

$a_t = 1$:<=> a at point of time t is stable at 1

$a_t = 0$:<=> a at point of time t is stable at 0

$a_t = p$:<=> a at point of time t is increasing

$a_t = n$:<=> a at point of time t is decreasing

$a_t = 1$:<=> $p(a_t \in \{0,p,1,n\}) \neq 1$

Having these "definitions" in mind it make sense to define disjunction conjunction and negation on FW as follows:

*	0	p	1	n	u
0	0	0	0	0	0
p	0	p	p	u	u
1	0	p	1	n	u
n	0	u	n	n	u
u	u	u	u	u	u

+	0	p	1	n	u
0	0	p	1	n	u
p	p	p	1	u	u
1	1	1	1	1	1
n	n	u	1	n	u
u	u	u	1	u	u

	-
0	1
p	n
1	0
n	p
u	u

Logic systems with n values are investigated in the literature and are not subject of this paper.

3. Ideal Five Valued Quasi Real Boolean Functions

Let $pr_t(a)$ be the projection of a signal at a point of time t.
$h:(S_T)^n \rightarrow S_T$ is called Ideal Five Valued Quasi Real Boolean Function
$:\Leftrightarrow$
$\exists h':FV^n \rightarrow FV: \forall a \in S_T : \forall t \in T: pr_t(h(a)) = h'(pr_t(a))$

4. The inertial edge triggered delay function

As typical example for a function describing timing phenomena the inertial edge triggered delay function is defined below.

4.1 A sex-tupel IED := (up,dn,i0,i1,ep,en) $\in N_0^6$ is called valid inertial edge triggered delay specification :<=> i0 < dn and i1<up.

By up we mean the delay of a positive transition, by dn the delay of a negative one. The minimal time a value 0 must be stable to be accepted is given by i0, while for value 1 this is given by i1. By ep we mean the longest allowed time interval during which a signal may have value up so such it causes an edge-triggering, en means the same for negative edges. In both cases the value 0 stands for "no edge triggering".

4.2 Let IED :=(up,dn,i0,i1,ep,en) be an inertial edge triggered delay specification, mied := max{up,dn}+max{i0,i1,ep,en{. With respect to IED the function iedf : S_T -> S_T is called inertial edge triggered delay function :<=>

$\exists iedf':FV^{mied} \rightarrow FV: \forall a \in S_T : \forall t \in T: pr_t(iedf(a)) = iedf'(pr_t(a),..,pr_{t-mied}(a))$

with

$$iedf'(pr_t(a),...,pr_{t-mied}(a)) = \begin{cases} pr'_{t-up}(a) & \text{if } pr_t(a) \in \{1,p\} \text{ or } pr_t(a)=u \text{ and } dn>= up \\ pr'_{t-dn}(a) & \text{if } pr_t(a) \in \{0,n\} \text{ or } pr_t(a)=u \text{ and } dn< up \end{cases}$$

with (for d∈{up,dn}, id∈{i0,i1}, ed∈{ep,en})

$pr'_{t-d}(a)=u$ <=>

1) $pr_{t-d}(a)$ =u or

2) $pr_{t-d}(a) \in \{0,1\} \wedge \exists t>t'>t-d>t''>t-mied$:

$pr_{t'}(a) \neq pr_{t-d}(a)$ and $pr_{t''}(a) \neq pr_{t-d}(a)$ and t'-t"<=id or

3) ed ≠ 0 and $pr_{t-d}(a) \in \{p,n\}$ and $\exists t>t'>t-d>t''>t-mied$:

$\forall t'>=t^+>=t''$: $pr_{t^+}(a)=pr_{t-d}(a)$ and t'-t">=ed

$pr'_{t-d}(a)=1$ <=>

4) $pr_{t-d}(a)=1$ and $pr'_{t-d}(a) \neq u$ or

5) ed ≠0 and $pr_{t-d} \in \{p,n\}$ and $\exists t>t'>t-d>t''=t-d-1$:

$pr_{t'}(a) \in \{0,1\}$ and $pr_{t''}(a) \in \{0,1\}$ and t'-t"<ed

$pr'_{t-d}(a)=0$ <=>

6) $pr_{t-d}(a)=0$ and $pr'_{t-d}(a) \neq u$ or

7) ed ≠0 and $pr'_{t-d}(a) \notin \{1,u\}$

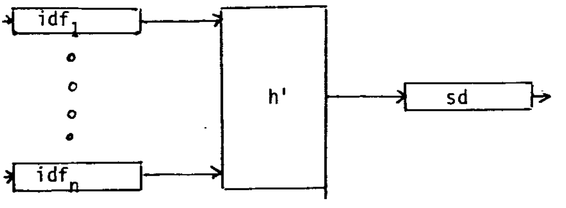
$pr'_{t-d}(a)=pr_{t-d}(a)$ otherwise.

The basis of this function is the linear translation of a signal by a simple delay. By this all the information to act inertialy and edge triggered is available. By inertia the delayed value is mapped on value u whenever it should be a stable one but remains not as long stable as requested by the inertia specification (2). The edge triggering maps a transition on value 1 at a single point of time if the "slope" of the transition is "high enough"(5). Too smooth edges are mapped on value u (3).

Under a pure delay function we understand a delay function that is defined with respect to a delay specification wher i0=0,i1=0,ep=0,en=0. Similary smoothing pure delay functions are defined. They decrease the slope by simply adding some additional values ep or en.

5) Five Valued Quasi Real Boolean Functions

Five Valued Quasi Real Boolean Functions now can be defined simply by composition of a smoothing pure delay function, an ideal Quasi Real Boolean Function an n inertial edge triggered delay functions. A formal definition is omitted in this abstract.
Such a FVQRBF as it is abbreviated may be imagined by the following picture:



6) Applications

When examining the above definitions one observes that a FVQRBF is defined with the aid of finite time intervals. Therefore it can be shown easyly that a FVQRBF can be modelled by a finite automaton of rather simple structure. By composing such automata a first approach for a high precision digital simulator is given. This simulator however would be very ineffective as a lot of state transitions of the automata would be trivial ones. Fortunately this concept may be mapped onto event oriented simulation method resulting in an effective high precision simulator.
High precision means:
- inertial delay with assignable delay and inertias
- potential different delays and inertias for different values
- edge sensitiv switching elements
- minimal slope and smoothing of transitions.

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