

MATHEMATICS TEACHERS SHOULD HAVE THE CONVICTION THAT EVERY INDIVIDUAL SHOULD BE TAUGHT MATHEMATICS

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Because of lack of time I must give my talk in a rather brief manner. But my suggestions can and should be discussed afterwards and in the next sessions.

We all want to improve mathematics education, and we think improving teacher education can help. But any progress in this matter has its natural limits.

- There is only a certain amount of time available for teacher education (on the whole and in each week).
- there is too little opportunity for teacher students to get acquainted with pupils and teaching at school while still at college.
- Teacher trainers in schools often tell the beginners to forget everything they have learned at college.
- Finally, in service, there are many tasks and difficulties inside as well as outside the classroom which have little to do with mathematics teaching, and as a result a lot of teachers tend to choose a rather comfortable way of teaching, which is not believed by teacher educators to always be the best.

Compared with these basic problems the possible effects of changes in the preparation of teachers in mathematics and mathematics education seem to be negligible. - Even if this is the case, we still should prepare our teacher students in the best way possible and, of course, we should think which way is the best. Furthermore, that statement is only partially true: We know that every mathematics teacher has a special socialization (with regard to his own subject, just as much as many non mathematics teachers have it) which develops at school and at college and has a noticeable influence on his professional life. This seems to me to be a possible starting point for improvements.

I do not plan to give a new list of mathematical or mathematics educational topics to be studied by prospective teachers. I also need not mention that

teacher students should be taught that mathematics should be taught along fundamental ideas, as a process, activity oriented, etc. Rather I want to discuss some problems in relation to the question of legitimation.

If asked, why every individual should be taught mathematics, we would think of some major reasons like:

1. mathematics for dealing with the physical (natural and technical) and social (e.g. economics) environment in everyday life and professional life,
2. mathematics for the training of cognitive abilities (e.g. spatial visualization) and for personality development (e.g. self-assuredness, willingness to tackle problems, reliability, etc),
3. mathematics as a cultural value (also: for fun, and: for its own sake).

It is my strong feeling that, of course, every prospective mathematics teacher, every mathematics teacher, and every mathematics teacher educator could name reasons similar to these, but that many of them are not really convinced that these reasons really hold. In fact, most of the members of the mathematics teaching community once decided to enter it, because

- they thought the chance of becoming a teacher in the first place would be better, and after becoming one the task would be easier, if they had mathematics as a subject, or
- at school they performed well in this subject, or
- they just love mathematics as such.

Any of these motives is honorable, but they do not suffice to teach this subject to pupils, most of whom, whether high or low achievers, have, or are likely to have, different personal interests in school and definitely outside school. Although we often will not be able to fully convey to our pupils why they should do mathematics at all and then why particular items, we should try, and, much more important, we ourselves must be convinced that they should. For this conviction reason no. 3 alone is much too weak. Reasons no. 1 and no. 2 are also needed.

As for reason no. 1: Many of our students haven't got any working experience outside school and college, which cannot be replaced by an intensive study of so called applied mathematics, physics or economics, because these subjects






are treated purely theoretically again, at least in their meaning to the students.

Our teacher student therefore should

- get a broad general education at school, before they specialize on a few subjects at college,
- get to know real professional life by working outside the educational system for some time, before becoming a teacher student,
- be made acquainted with many genuine examples of applied mathematics accessible and (possibly) relevant to that vast majority of people who will never belong to the community of mathematicians and mathematics teachers.

This last point is the teacher educators' task. They should have had a similar career, and they should be able to teach applications - not necessarily systematically, but extensively. To accomplish this, we need much more knowledge about the shapes of all kinds of mathematics in ordinary people's everyday lives and professions. This is a truly didactical problem which cannot be settled by just asking vocational instructors or analyzing aptitude tests for vocational training, because they often reflect an idea of mathematics which we want to overcome: mathematics as an instrument for selection rather than for personal enrichment.

A. Schreiber and I have done some work about the connections between geometry and the real world and have elaborated a lot of concrete examples, e.g.

- the shape of bolt nuts  ?  ?  ?  !  !
- the practical use of the helix (for transportation, for uncorking bottles, for all kinds of caps and tops, etc.).
- the polyhedral structure of the leather football



I am not aiming at a so called polyvalent teacher education (where teacher students shall be qualified for a second profession at the same time), but at a broader teacher education, with mathematics still as a central subject for mathematics teachers (and possibly some mathematics for every teacher), hoping that teachers forward this education to the students.

This brings me to reason no. 2: Teachers must be interested not only in the subject they teach, but definitely also in the individuals they teach. They should understand them and be able to relate to their feeling and thinking. Of course, this is not a problem of mathematics education only, but of any education.

Prospective mathematics teachers usually get to know a lot about mathematics, and, hopefully, about mathematics teaching, and methods of how to treat mathematical issues, etc. But these methods usually fit only into ideal, non-realistic classroom situations (whether instruction is individualized, or not). The fundamental failure of contemporary teacher education lies in prospective teachers learning to prescribe to abstract students ideal ways of thinking, instead of learning to appreciate real students' actual ways of thinking and feeling. Consequently, a lot of teachers assume as a general principle that students (if they are not too stupid) have the right concepts, methods, knowledge, etc., especially when needed for a following instructional unit; and many teachers and teacher educators have been quite surprised about the results of, e.g., Kathleen Hart's research on secondary students' understanding of mathematics.

So our teacher student should

- have a good sense for other people's thinking and feeling, which could also have been developed at school, at least partially,
- widen their social horizon by spending some time working outside the educational system,
- be made acquainted with many genuine examples of students' typical reactions to mathematics instruction and learn how to interpret these reactions.

Again, all these points also concern teacher educators, and the last one is their special task.

One way to create such examples is to give a test to many pupils in order to find certain regularities and then ask some pupils personally about their reasoning. For example, H. Winter and I investigated the mathematical competencies in the lives of ten-year-olds. One question was: which of the fol-

lowing patterns will most likely occur, when rain starts falling on two paving stones?



15 % chose the second; they had answered the "wrong" question, namely: which pattern is the clearest?

In a second step, teacher students should analyze real mathematics instruction, and find didactical faults (unsuitable introductory examples, lacking interrelation between modes of representation, insufficient formation of basic concepts, etc.) and their short-, middle-, and longterm effects. There remains, of course, a fundamental problem: How do we get such examples of lessons or units? Which teacher thinks that his instruction could provide them? Who wants to be responsible for intentionally bad instruction? - Our teacher students' first lessons seem to be inappropriate. Maybe we have to record a few standard examples on video tape, which we come across more or less by chance.

Then, in a third step, the social and communicational dimension of classroom instruction should be taken into consideration. Of course, this is the most difficult part not only in teacher education but also in theory building and in research. But teacher students should at least become aware of this dimension.

In my own work I have not yet realized steps no. 2 and no. 3.

From my experience I can tell that there are always a few teacher students who are not interested in analyzing geometrical ideas in the real world or pupils' statements, but who believe strongly in written curricula. Those are the students at whom the title of this talk is directed.

I do not plead for a relapse into non-academical teacher education. On the contrary, all suggested activities can only be successful on the basis of a thorough study of mathematics, as well as educational and related theories, like psychology, etc.